



Unconventional pairing in three-dimensional topological insulators with warped surface state

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Topological insulator in a nutshell

...a new state of matter that has been predicted and discovered!

□ Bulk is insulating; edge (2D)/ surface (3D) a very good conductor.

□ Important ingredient: spin-orbit coupling:

opposite force for opposite spins.

□ Topological invariant is insensitive to any continuous deformation of Hamiltonian (**topological protection**): disorder, geometry, weak interactions, etc...

Examples:

□ **2D**: HgTe/CdTe; **3D**: Bi₂Se₃, Bi₂Te₃, Sb₂Te₃, TlBiSe₂, Bi₂Te₂Se.

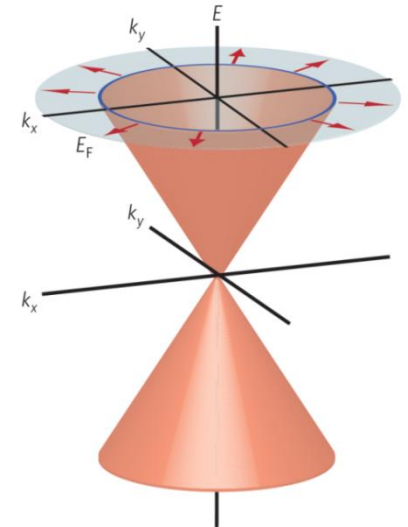
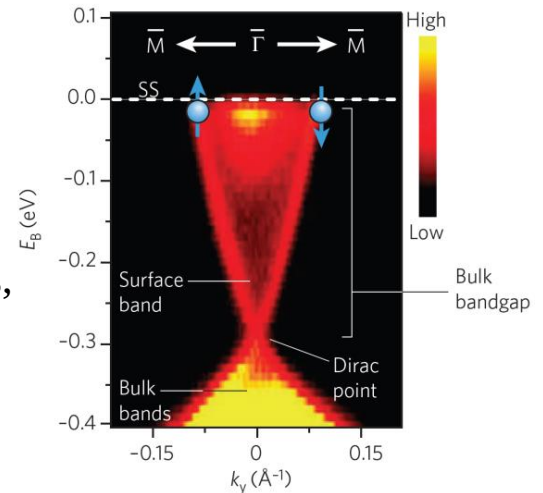
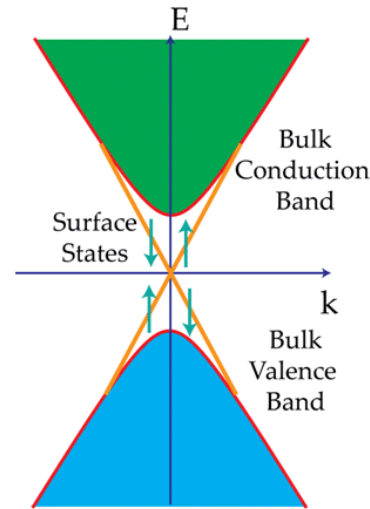


2D

Theo1: C.L. Kane and E.J. Mele, *PRL* 95, 226801 (2005)

Theo2: B.A. Bernevig et al., *Science* 314, 1757 (2006)

Exp: M. König et al., *Science* 318, 766 (2007)



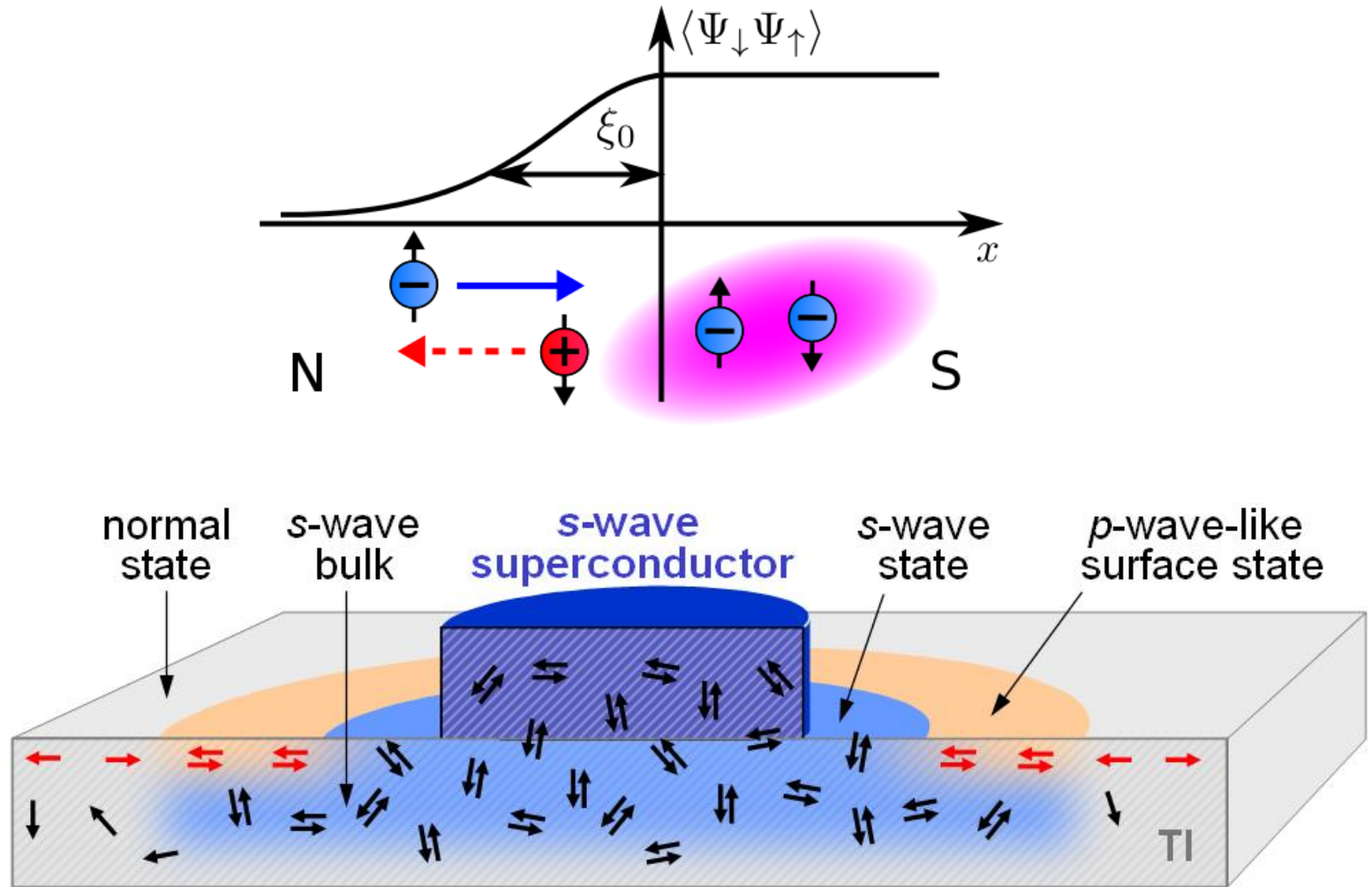
3D

Theo: L. Fu, et al., *PRL* 98, 106803 (2007)

Exp1: Zhang H. et al., *Nat. Phys.* 5, 438 (2009)

Exp3: S. Takafumi et al., *PRL* 105, 136802 (2010)

S/TI proximity effect



J. Shen et al. (2013)

Symmetry classification of induced pair potential

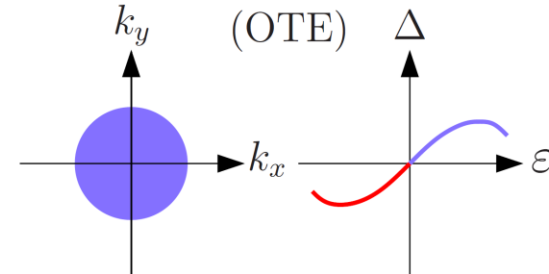
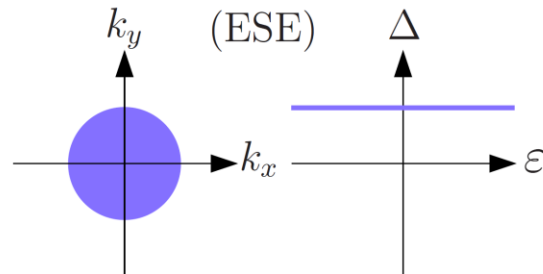
Fermi-Dirac statistics

Symmetry of pair wave functions:

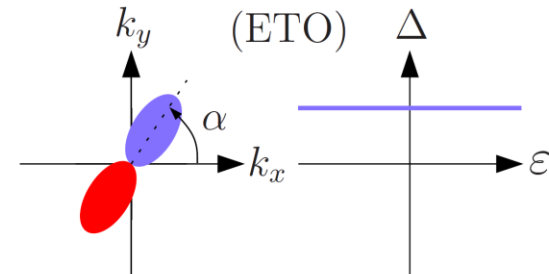
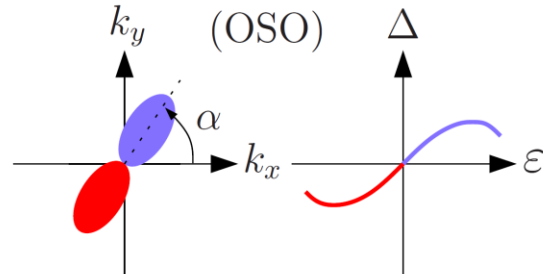
$$\mathbf{k} \otimes \sigma \otimes \omega = \text{odd}$$

	$F_{\sigma\sigma'}(\omega, k) = -F_{\sigma'\sigma}(-\omega, -k)$		
	$\omega \rightarrow -\omega$	$\sigma \leftrightarrow \sigma'$	$k \rightarrow -k$
<i>ESE</i>	+	-	+
<i>OSO</i>	-	-	-
<i>ETO</i>	+	+	-
<i>OTE</i>	-	+	+

Even frequency-singlet-even parity Odd frequency-triplet-even parity



Odd frequency-singlet-odd parity Even frequency-triplet-odd parity



J. Linder et al., PRB (2008)

Majorana fermion

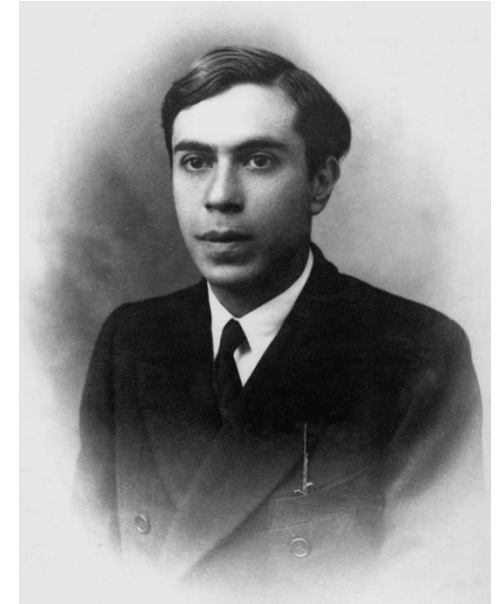
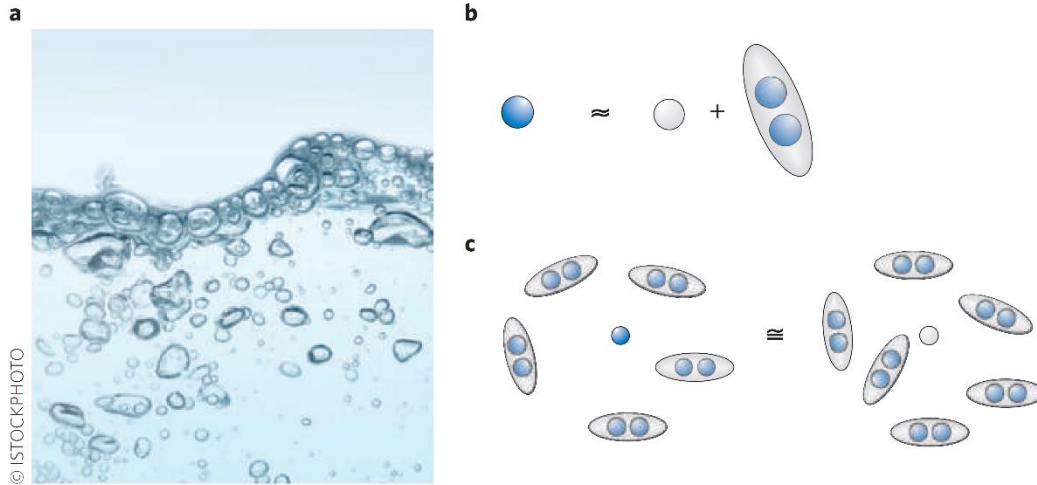
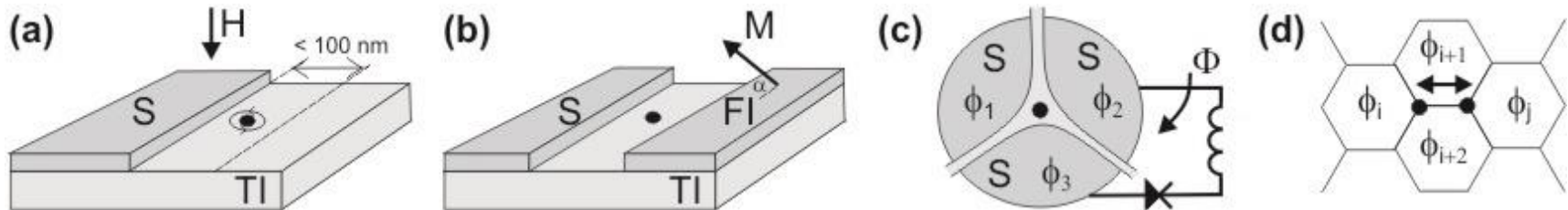


Figure 1 | Antimatter matters in the solid state. **a**, A familiar concept in solid-state physics, holes are bubbles of missing electrons in the Fermi sea of the electronic spectrum, behaving like positively charged electrons. **b**, In a superconductor, the properties of electrons (blue) and holes (grey) are drastically modified by their interaction with the surrounding sea of Cooper pairs; a hole can attract or bind to a Cooper pair, and acquire negative charge. **c**, More importantly, Cooper pairs cluster around holes and thin out around electrons, in such a way that no rigorous distinction between them remains.

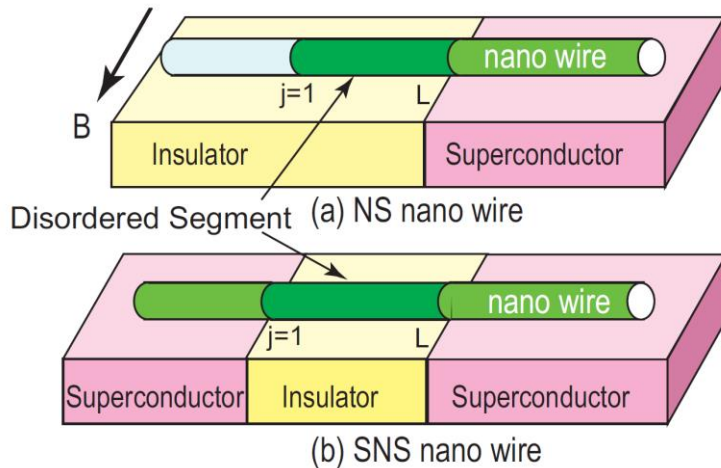
F. Wilczek, Nat. Phys. (2009)



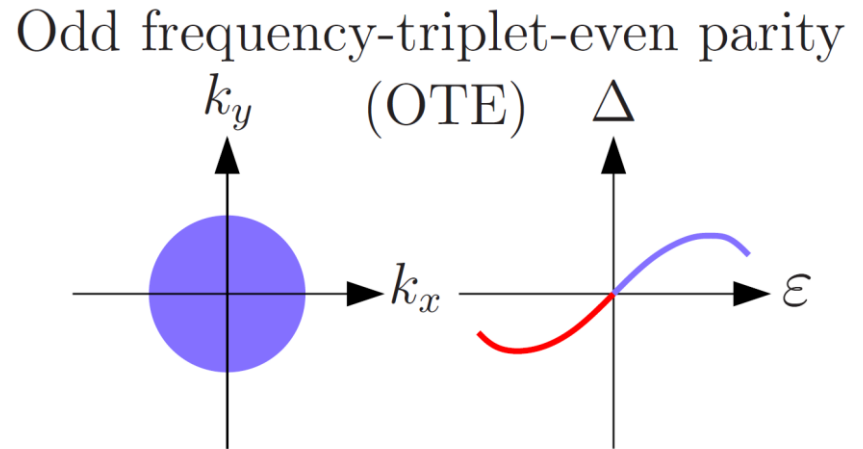
Fu, Kane, PRL (2008)

Tanaka, Yokoyama, Nagaosa, PRL (2009)

Odd-frequency and Majorana



Asano, Tanaka, PRB (2013)



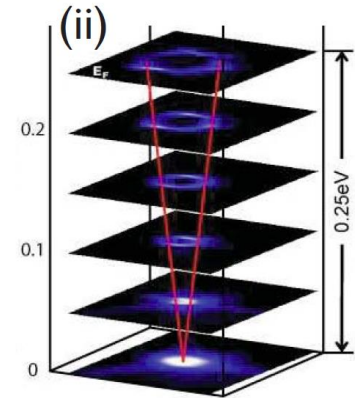
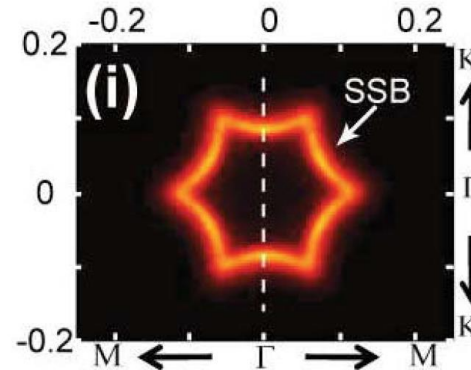
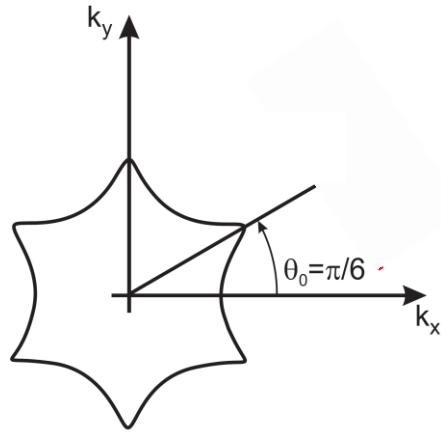
The physics behind the anomalous transport can be understood in terms of the odd-frequency Cooper pairing. We conclude that Majorana fermions and odd-frequency Cooper pairs in solids are two sides of a same coin.

Hexagonal warping in 3D topological insulators

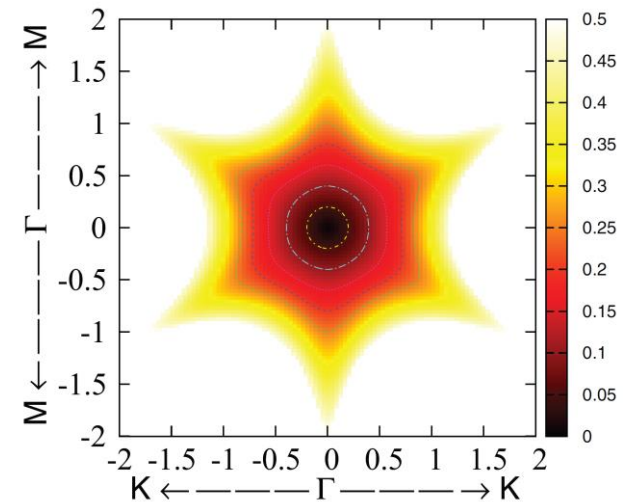
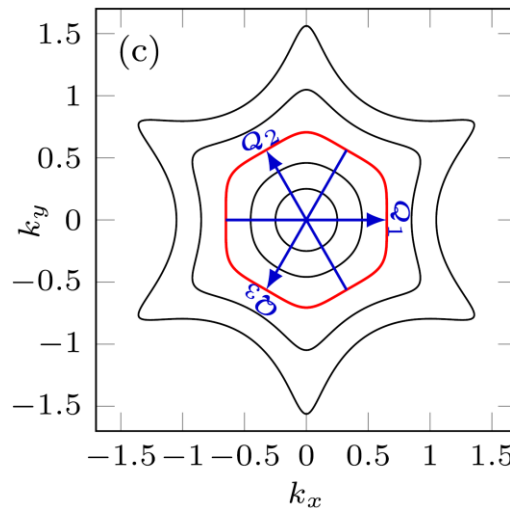
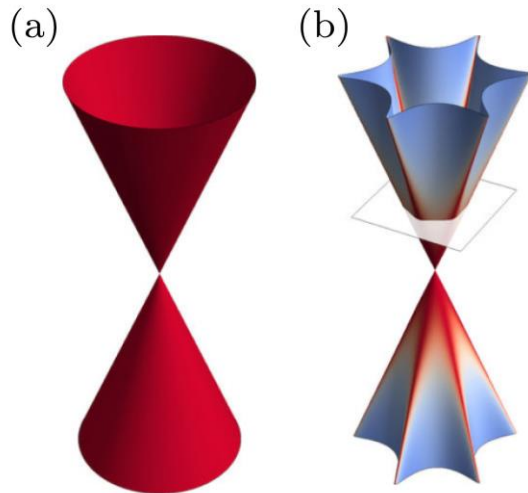
$$\hat{H}(\mathbf{k}) = -\mu + v(k_x \hat{\sigma}_y - k_y \hat{\sigma}_x) + \hat{H}_w(\mathbf{k})$$

$$\hat{H}_w(\mathbf{k}) = \lambda k^3 \cos(3\theta) \hat{\sigma}_z$$

Fu, PRL (2009)



Chen et al., Science (2009)



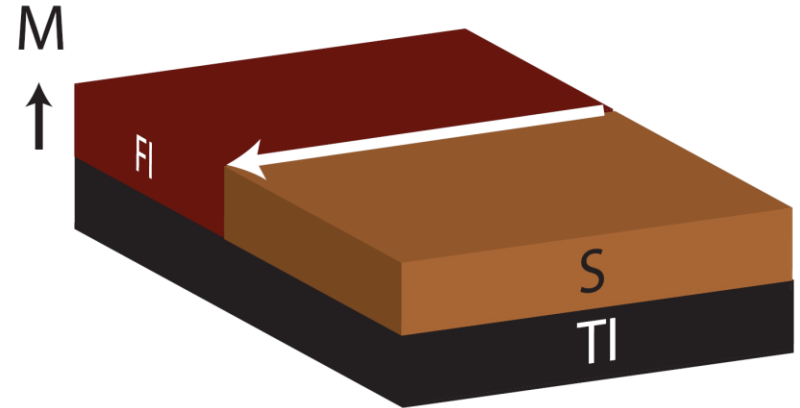
Mendle, Kotetes, Schon, PRB (2015)

Li, Carbotte, PRB (2013)

Model: S/FI hybrid on TI surface

Bogoliubov – de Gennes – Dirac Hamiltonian

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$



Green's function (Nambu + spin space)

$$[E - \check{H}_S(\mathbf{k})] \check{G} = \check{1} \quad \check{G} = \begin{pmatrix} \hat{G}_{ee} & \hat{G}_{eh} \\ \hat{G}_{he} & \hat{G}_{hh} \end{pmatrix}$$

Anomalous Green's function

Bergeret, Volkov, Efetov, RMP (2005)

$$\hat{G}_{eh} = i(f_0 \hat{\sigma}_0 + f_x \hat{\sigma}_x + f_y \hat{\sigma}_y + f_z \hat{\sigma}_z) \hat{\tau}_y$$

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & & & \\ (\uparrow\downarrow - \downarrow\uparrow) & \text{singlet} & \text{triplet} & \text{triplet} & \text{triplet} & & (\uparrow\downarrow + \downarrow\uparrow) \\ & & (\uparrow\uparrow - \downarrow\downarrow) & (\uparrow\uparrow + \downarrow\downarrow) & & & \end{array}$$

No warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x)$$

$$\hat{G}_{\text{eh}} = i(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z)\hat{\tau}_y$$

Anomalous Green's function symmetry, Z is even in E and k

$$f_0 = \frac{\Delta}{Z} (E^2 + M^2 - \mu^2 - \Delta^2 - v^2 k^2), \quad (\uparrow\downarrow - \downarrow\uparrow) \quad \textbf{ESE}$$

$$f_x = \frac{2\Delta}{Z} kv [\mu \sin(\theta) + iM \cos(\theta)], \quad (\uparrow\uparrow - \downarrow\downarrow) \quad \textbf{ETO}$$

$$f_y = -\frac{2\Delta}{Z} kv [\mu \cos(\theta) - iM \sin(\theta)], \quad (\uparrow\uparrow + \downarrow\downarrow) \quad \textbf{ETO}$$

$$f_z = \frac{2\Delta}{Z} EM. \quad (\uparrow\downarrow + \downarrow\uparrow) \quad \textbf{OTE}$$

No warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x)$$

$$\hat{G}_{\text{eh}} = i(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z)\hat{\tau}_y$$

Anomalous Green's function symmetry, Z is even in E and k

$f_0 = \frac{\Delta}{Z} (E^2 + M^2 - \mu^2 - \Delta^2 - v^2 k^2),$	$(\uparrow\downarrow - \downarrow\uparrow)$	ESE
$f_x = \frac{2\Delta}{Z} kv [\mu \sin(\theta) + iM \cos(\theta)],$	$(\uparrow\downarrow - \downarrow\uparrow)$	ETO
$f_y = -\frac{2\Delta}{Z} kv [\mu \cos(\theta) - iM \sin(\theta)],$	$(\uparrow\uparrow + \downarrow\downarrow)$	ETO
$f_z = \frac{2\Delta}{Z} EM.$	$(\uparrow\downarrow + \downarrow\uparrow)$	OTE

Finite warping

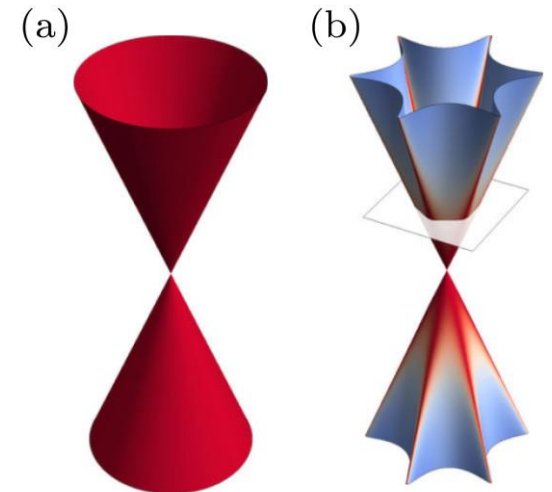
$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix} \quad \hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_w(\mathbf{k})$$

$$\hat{G}_{\text{eh}} = i \left(\underset{\substack{\text{ESE} \\ \text{OSO}}}{f_0\hat{\sigma}_0} + \underset{\substack{\text{OTE} \\ \text{ETO}}}{f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z} \right) \hat{\tau}_y$$

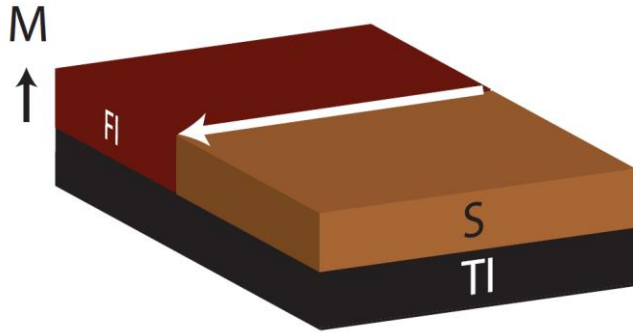
Anomalous Green's function z-component $(\uparrow\downarrow + \downarrow\uparrow)$

$$\begin{aligned} f_z &= f_z^- + f_z^+, \\ f_z^- &= EMF_{\text{even}} - \mu\lambda k^3 \cos(3\theta) F_{\text{odd}}, \\ f_z^+ &= EMF_{\text{odd}} - \mu\lambda k^3 \cos(3\theta) F_{\text{even}}. \end{aligned} \quad \begin{matrix} \text{OTE} \\ \text{ETO} \end{matrix}$$

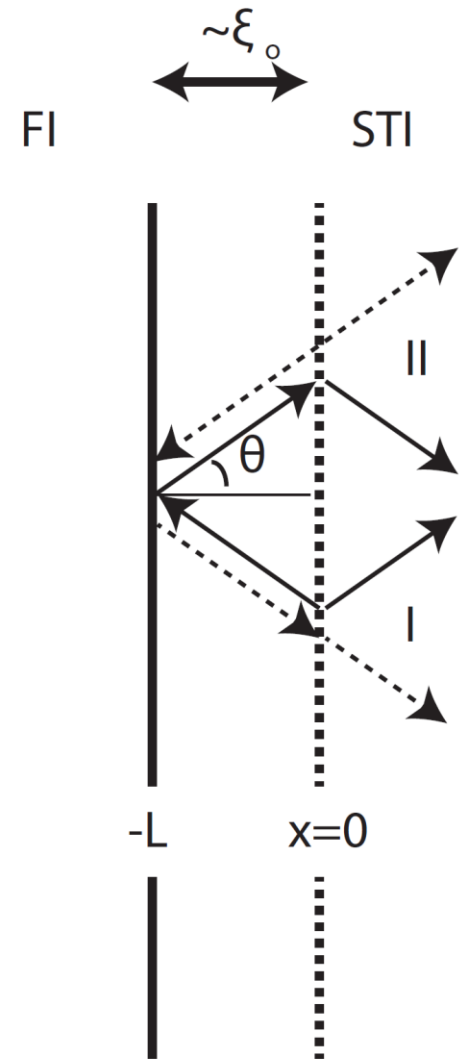
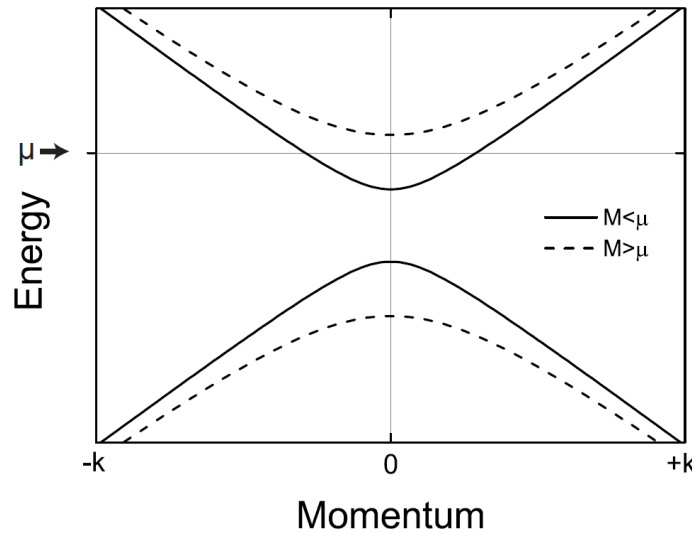
$$\theta_n = \pi/6 + \pi n/3$$



Majorana fermion realization



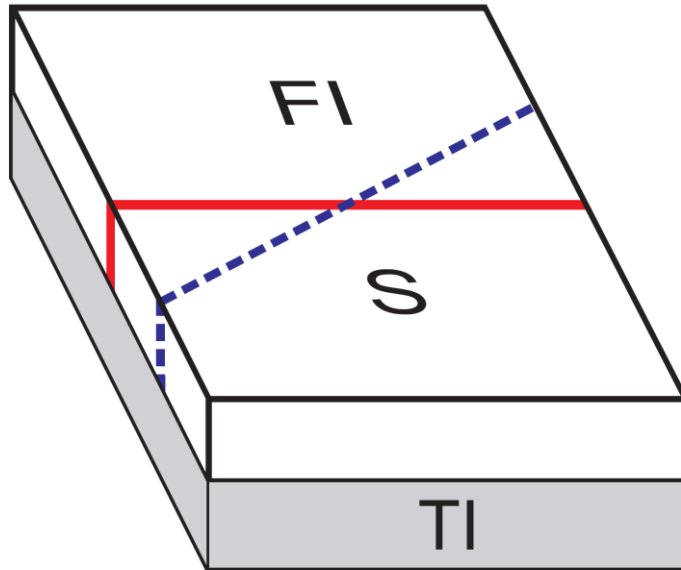
$$\frac{E}{\Delta} = -\sin(\theta) \\ = -k_y/|k|.$$



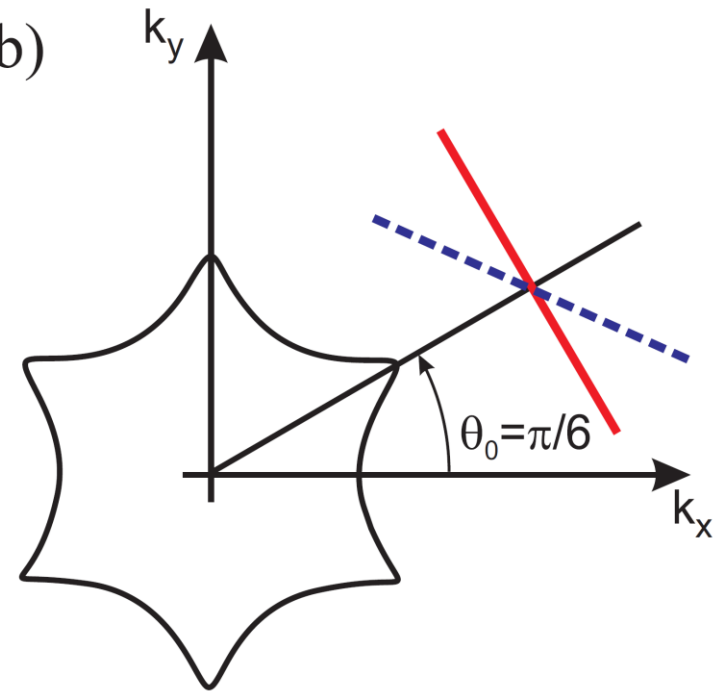
Snelder, Golubov, Asano, Brinkman (2015)

Majorana fermion and warping

a)



b)



Vasenko, Golubov, Silkin, Chulkov, arXiv:1606.00905 (2016)

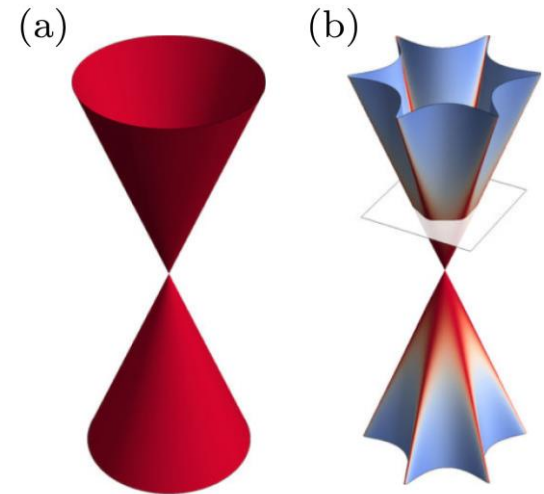
Spontaneous currents



$$\hat{H}_M(\mathbf{k}) = -\mu + v(k_x \hat{\sigma}_y - k_y \hat{\sigma}_x) + \lambda k^3 \cos(3\theta) \hat{\sigma}_z + M \hat{\sigma}_z$$

$$\hat{H}(\mathbf{k}) = -\mu + vk_x \hat{\sigma}_y + \hat{\sigma}_x M_x + \hat{\sigma}_y M_y + \hat{\sigma}_z M_z$$

Nesterov, Houzet, Meyer, arXiv:1512.03042 (2016)



Review

- We discuss singlet to triplet mixing in proximized 3D topological insulators with warped surface state
- We establish the selection rule for Majorana Fermion realization in S/FI structures formed on the surface of the TI: S/FI boundary should be properly aligned with respect to the snowflake contour.
- Spontaneous currents in S/TI hybrids.

Thank you!