

Fluctuations in quasi-1-dimensional superconductors

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Outline

1. Introduction

Fluctuations in superconductors
Quantum Phase Slip concept

2. Impact of phase slips

$R(T)$

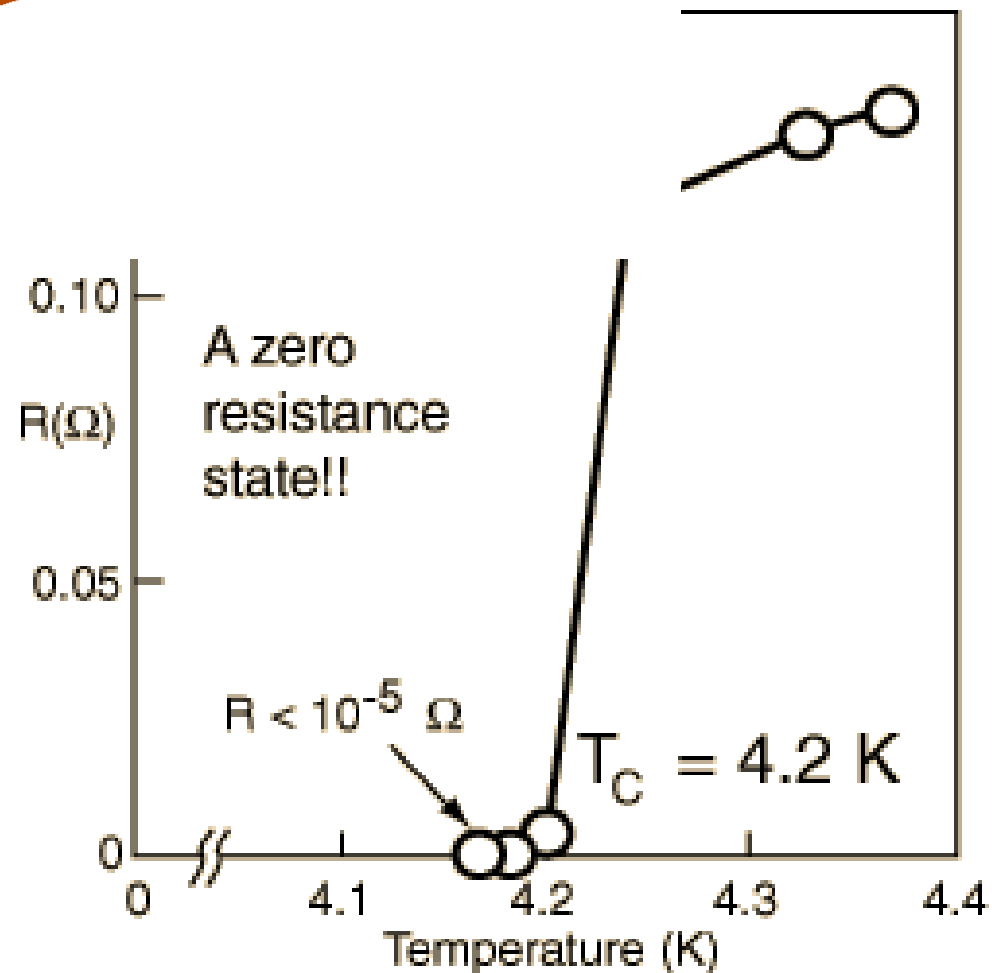
Persistent currents

3. Impact of modulus (?) fluctuations

DOS and superconducting gap

RITJ transition

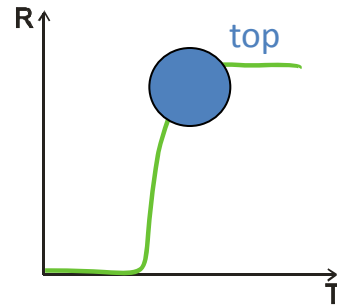
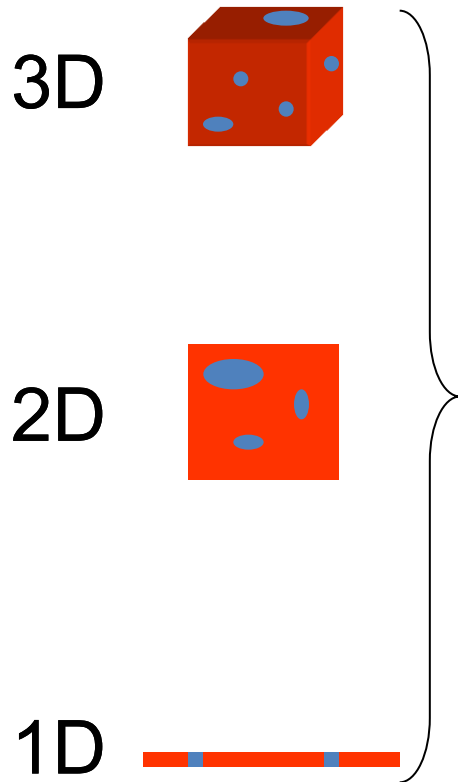
H. K. Onnes,
Commun. Phys. Lab.12,120, (1911)



Fluctuations vs. system dimensionality

For a superconductor dimensionality is set by the temperature - dependent coherence length $\xi(T)$.

N normal metal
S superconductor



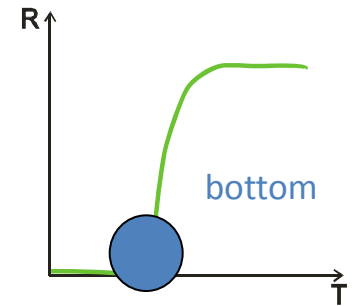
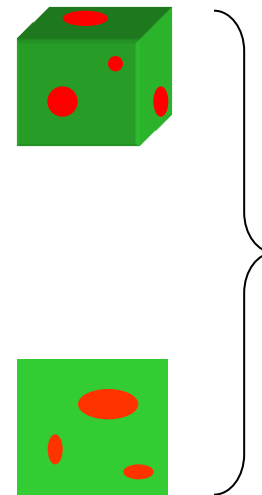
S inclusions reduce the total system resistance $\rightarrow$

rounded top

$$\sigma_{\text{FLUCT}} \sim (T - T_c)^{-(2-D/2)}$$

(Aslamazov – Larkin)

Topic of the talk



no contribution of N inclusions: normal current is shunted by supercurrent $\rightarrow$

abrupt bottom



N inclusions block the supercurrent $\rightarrow$

rounded bottom

(Langer – Ambegaokar)

Fluctuations in a 1D superconductor

$$\xi(T)$$


Long 1D wire of cross section σ



$$v\sigma < \xi(T) \ll L$$

If the wire is infinitely long, there is always a finite probability that in some fragment(s) the magnitude of the order parameter instantly becomes zero and the phase changes by 2π

The minimum length the superconductivity can be destroyed is the coherence length $\xi(T)$

The minimum energy corresponds to destruction of superconductivity in a volume $\xi(T) \sigma$:
 $\Delta F = B_c^2 \xi(T) \sigma$, where $B_c(T)$ is the critical field

In the limit rare events the probability of the process $P(T) \sim \exp(-\Delta F / \mathcal{E})$

Thermal activation: $\mathcal{E} \sim k_B T$.
Important at $T \rightarrow T_c$.

Quantum: $\mathcal{E} \sim \Delta$.
Weak temperature dependence,
exist even at $T \rightarrow 0$

In current state the particular manifestation of a quantum fluctuation when magnitude of the order parameter momentarily nulls and phase changes by $\pm 2\pi$ is often called “phase slip”

Thermal activated phase slips

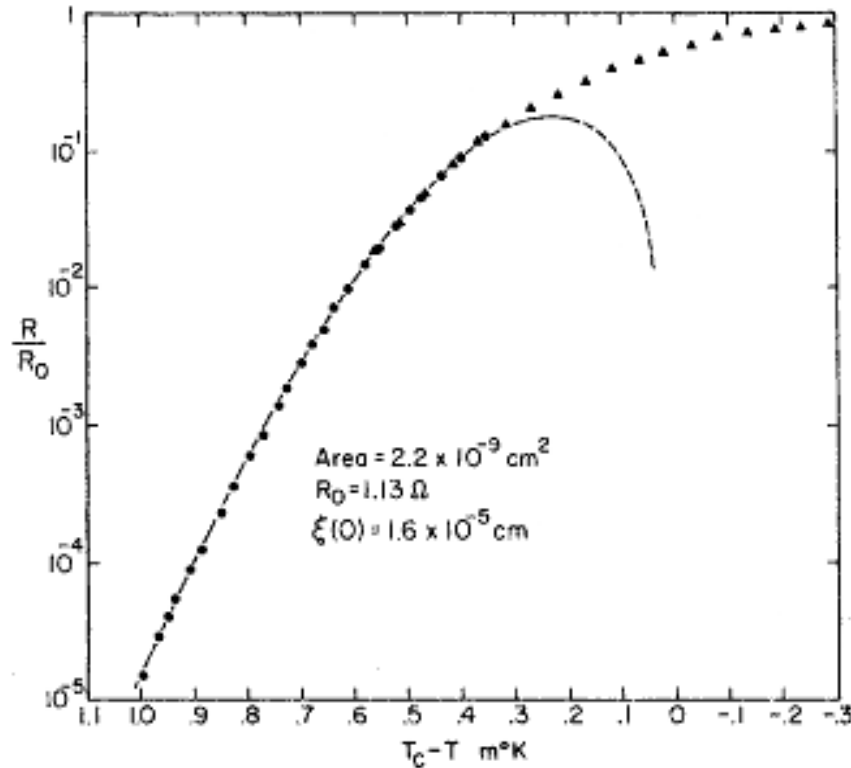
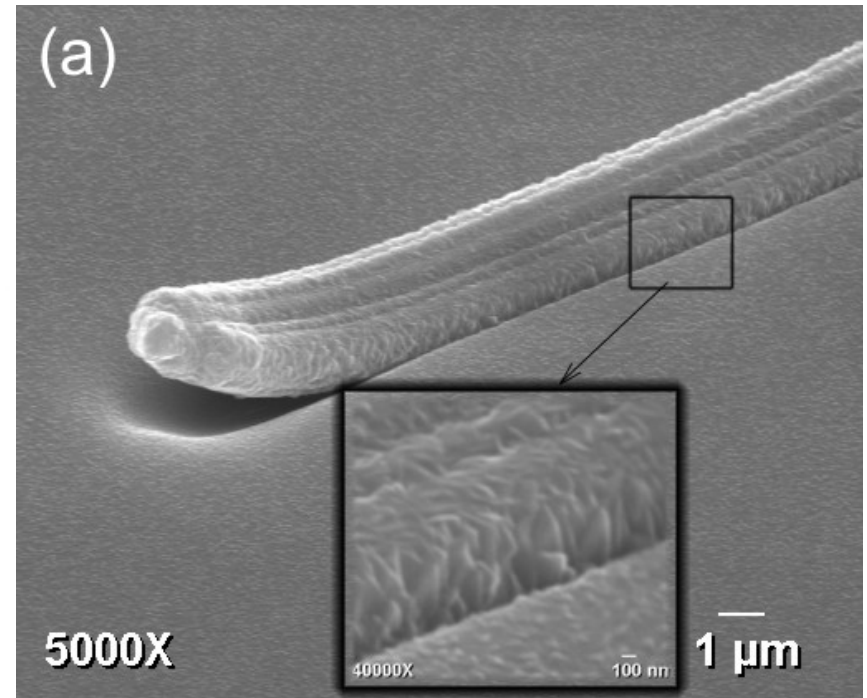


FIG. 3. Total resistance of tin crystal as a function of temperature. Closed circles, measured with superconducting femtovoltmeter; closed triangles, measured with nanovoltmeter. Line, two-parameter fit to M-H theory.



Tin whisker: $\sqrt{\sigma_{\text{eff}}} \sim 1 \mu\text{m}$

Experiment:

*J. E. Lukens, R.J. Warburton, W. W. Webb, Phys. Rev. Lett. **25**, 1180 (1970)*

*R. S. Newbower, M.R. Beasley, M. Tinkham, Phys. Rev. **B 5**, 864, (1972)*

The results have been confirmed on multiple 'not too thin' samples

QPS contribution

Full model (G-Z) $\longrightarrow$

A. Zaikin, D. Golubev, A. van Otterlo, and G. T. Zimanyi, PRL 78, 1552 (1997)

A. Zaikin, D. Golubev, A. van Otterlo, and G. T. Zimanyi, Uspexi Fiz. Nauk 168, 244 (1998)

D. Golubev and A. Zaikin, Phys. Rev. B 64, 014504 (2001)

In the limit $R(T) \ll R_N$ QPSs are activated at a rate:

$$\Gamma_{QPS} = E_{QPS} / \hbar = \Delta_0 [(R_Q L^2) / (R_N \xi^2)] \exp(-S_{QPS})$$

$$R(T) = b \frac{\Delta_0(T) S_{QPS}^2 L}{\xi(T)} \exp\{-2S_{QPS}\}$$

where $S_{QPS} = A \cdot [(R_Q / \xi) / (R_N / L)]$, $R_Q = \hbar / 4e^2 = 6.45 \text{ k}\Omega$, Δ = energy gap,

R_N = normal state resistance, L is wire length, $\xi = 0.85(\xi_0 L)^{1/2}$ is coherence length

$A \approx b \approx 1$ are numerical parameters.

Essentially, there is only one fitting parameter: $A \approx 1$.

For a dirty limit superconductor: $\Gamma_{QPS} \sim \exp(-A' \sigma T_c^{1/2} / \rho_N)$, where σ is the wire diameter and ρ_N is the normal state resistivity.

Materials with low T_c and high resistivity ρ_N are of advantage!

Existing experiments on QPS

J.M Graybeal, P. M. Mankiewich, R. C. Dynes, M. R. Beasley,
Phys. Rev. Lett. 59, 2697 (1987)

N. Giordano, et.al. PRL 61, 2137 (1988);
Phys. Rev. Lett. 63, 2417 (1989); Phys. Rev. B
41, 6350 (1990); Phys. Rev. B 43, 160 (1991);
Physica B 203, 460 (1994)

A. V. Herzog, P. Xiong, F. Sharifi, and
R. C. Dynes
PRL, 76, 668 (1996)

A. Bezryadin, C. N. Lau and M.
Tinkham, Nature 404, 971 (2000)

C. N. Lau, N. Markovic, M. Bockrath,
A. Bezryadin, and M. Tinkham, Phys.
Rev. Lett. 87, 217003 (2001)

MoGe nanowires

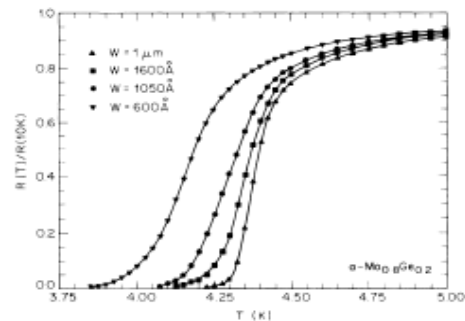
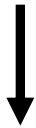


FIG. 1. Resistive transitions for a set of four wires with different widths w upon a single film of thickness 50 Å. Note the systematic decrease in T_c with decreasing sample width.



clean systems.²⁰ However, in this case one expects the resistance to fall off exponentially with temperature below T_c , which does not agree with the results of our

we may only conclude that the superconducting fluctuations are enhanced as the disordered 1D limit is approached.

Nanowires of type-I superconductors

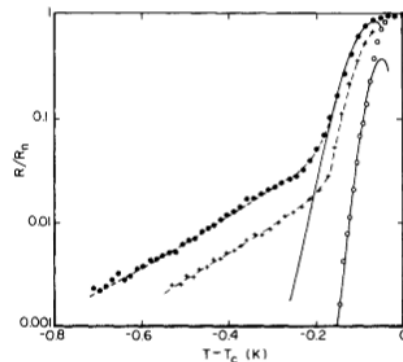


FIG. 1. Resistance, normalized by the normal state value, as a function of temperature for three In wires; the sample diameters were 410 Å (●), 505 Å (+), and 720 Å (○). The solid curves are fits to the thermal activation theory (1), while the dashed curves are fits to (7). Where the solid and dashed curves overlap, only the former is shown for clarity. The samples had lengths of 80, 150, and 150 μm, and normal-state resistances of 5.7, 7.1, and 1.2 kΩ, where we list the values in order of sample size, with the value for the smallest sample first.

in situ grown granular metal wires

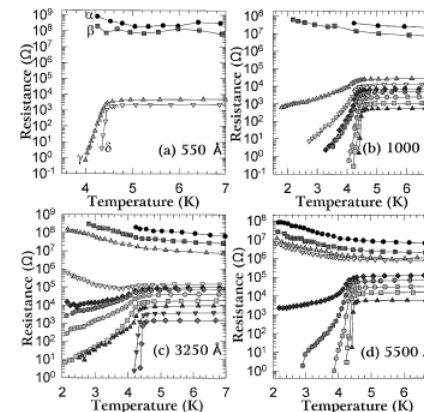


FIG. 1. The strong to weak localization transition for a series of Sn wires of increasing widths. Each set of data exhibits a gap in its resistance where no stable wire is obtained. (a) 550 Å wide: curve $\alpha \sim 78$ Å thick; $\beta \sim 81$ Å thick; γ approximately same thickness as curve β ; $\delta \sim 107$ Å thick. (b) 1000 Å wide. (c) 3250 Å wide. (d) 5500 Å wide.

MoGe film on top of a carbon nanotube

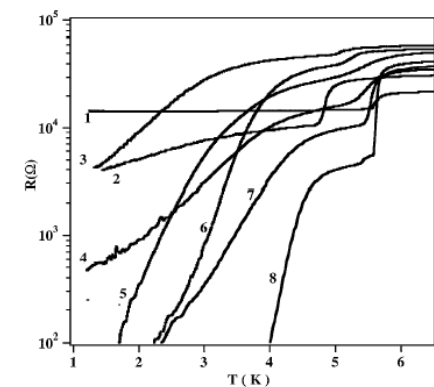
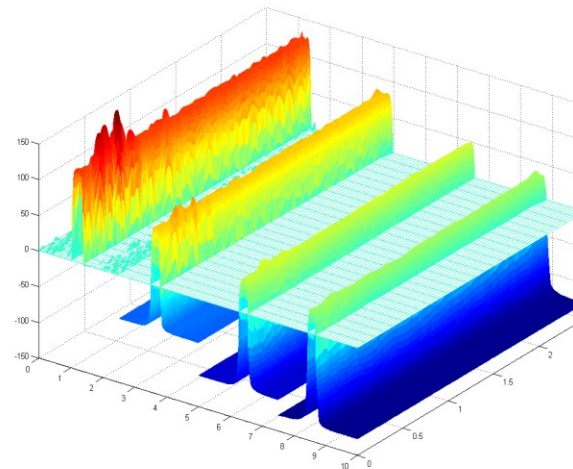
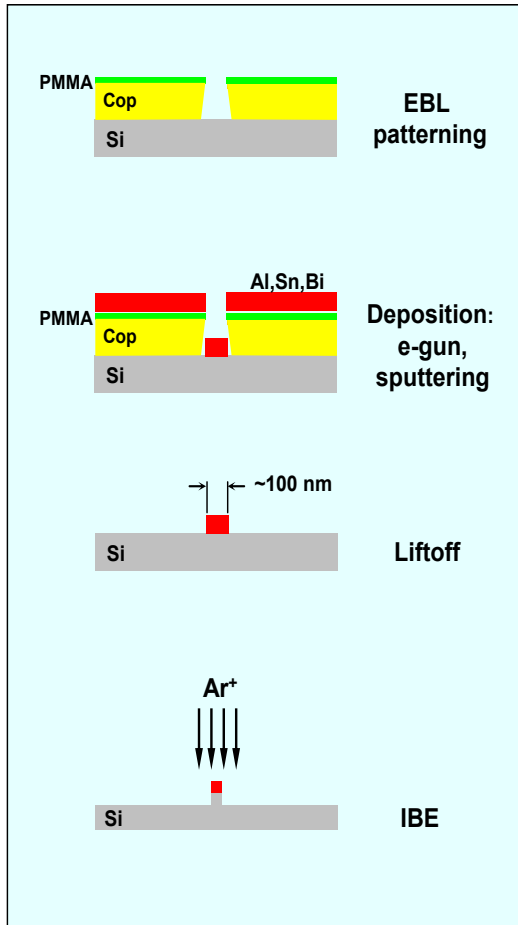


FIG. 1. Resistances as a function of temperature for eight different samples. The samples' normal state resistances and lengths are 1: 14.8 kΩ, 135 nm; 2: 10.7 kΩ, 135 nm; 3: 47 kΩ, 745 nm; 4: 17.3 kΩ, 310 nm; 5: 32 kΩ, 730 nm; 6: 40 kΩ, 1050 nm; 7: 10 kΩ, 310 nm; 8: 4.5 kΩ, 165 nm.

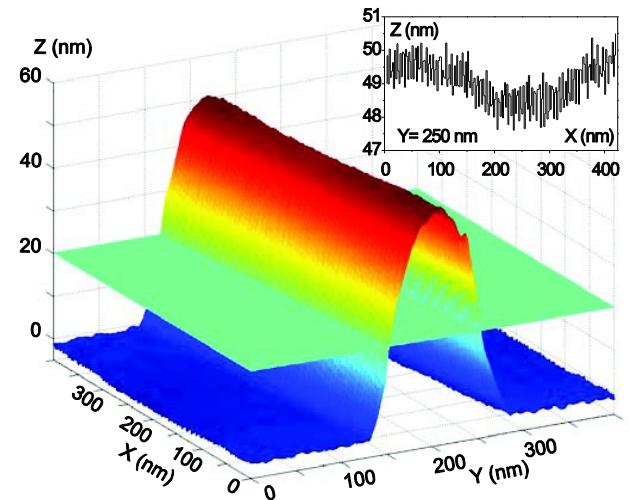
More systematic study is required!

1D samples: fabrication & shape control

Objective: to enable measurements of the same nanowire with progressively reduced diameter



Evolution of Al nanowire after several sessions of ion milling.



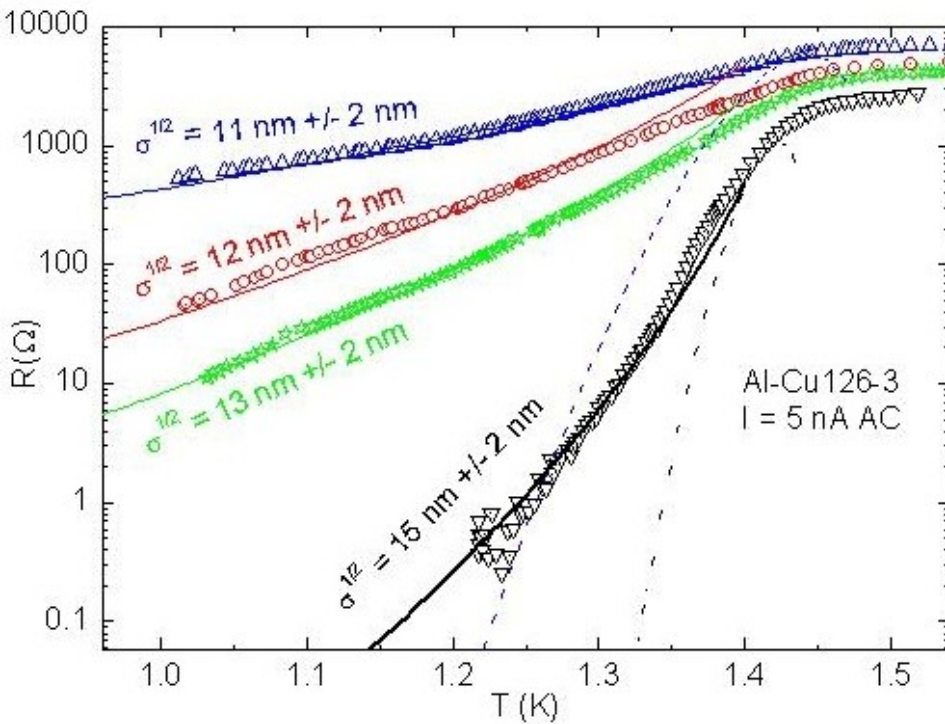
AFM image of a typical Ti nanowire. Inset shows the surface roughness ± 1 nm.

Ion beam provides polishing and gradual reduction of the cross-section

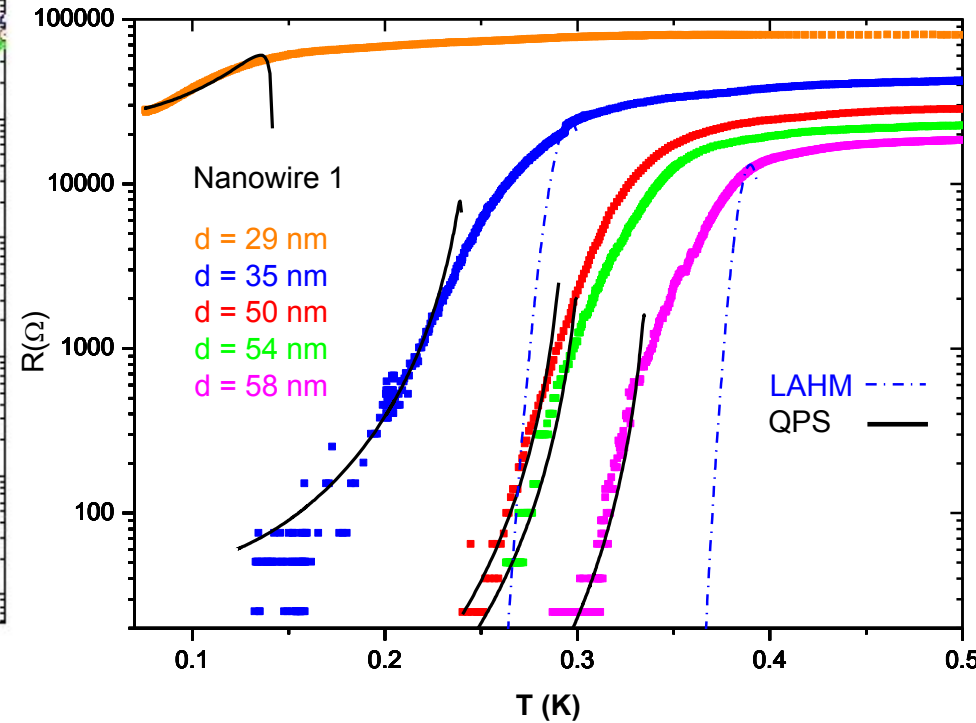
Penetration depth of 1 keV Ar^+ ions inside metal matrix < 2 nm

Manifestation of QPS: broadening of R(T) transition

Aluminium



Titanium



Dashed lines: thermally activated fluctuations (LAHM) at $T \sim T_c$

Solid lines: quantum fluctuations (Golubev – Zaikin) model at $T \ll T_c$

M. Zgirski, K.-P. Riikonen, V. Touboltsev, and K. Arutyunov. // **Nano Letters** 5, 1029--1033 (2005).

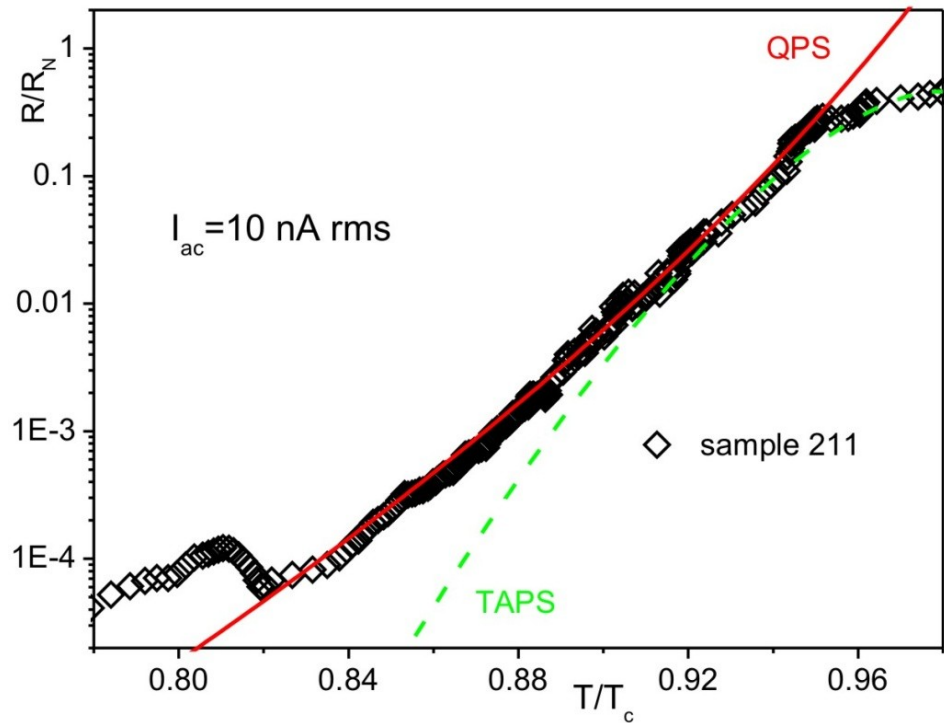
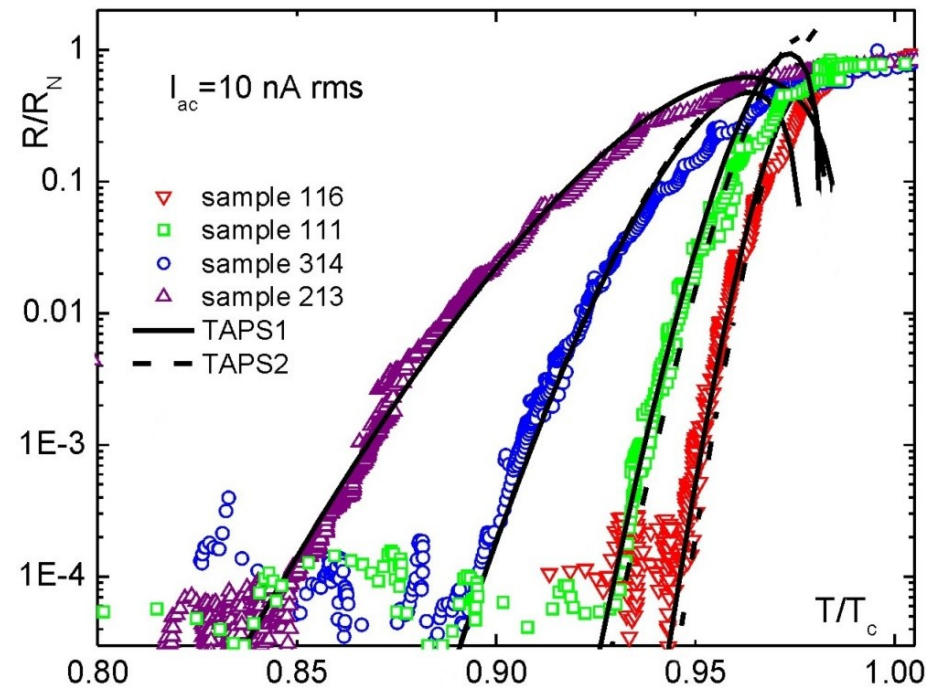
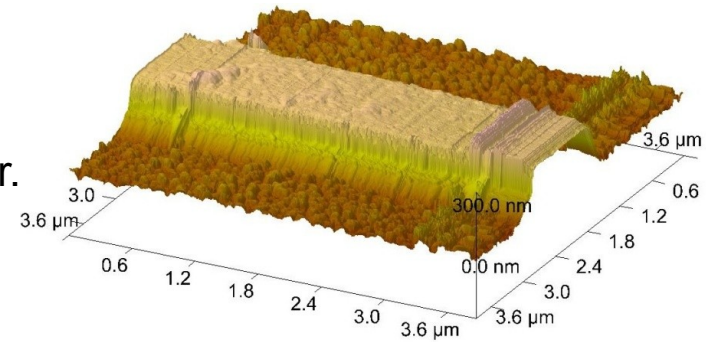
M. Zgirski, K.-P. Riikonen, V. Touboltsev and K. Yu. Arutyunov.// **PRB** 77, 054508-1 -- 054508-6 (2008).

J. S. Lehtinen, T. Sajavaara, K. Yu. Arutyunov, M. Yu. Presnjakov, and A. S. Vasiliev // **PRB** 85, 094508 (2012).

NbN nanowires

NbN is a highly disordered (= highly resistive) superconductor.

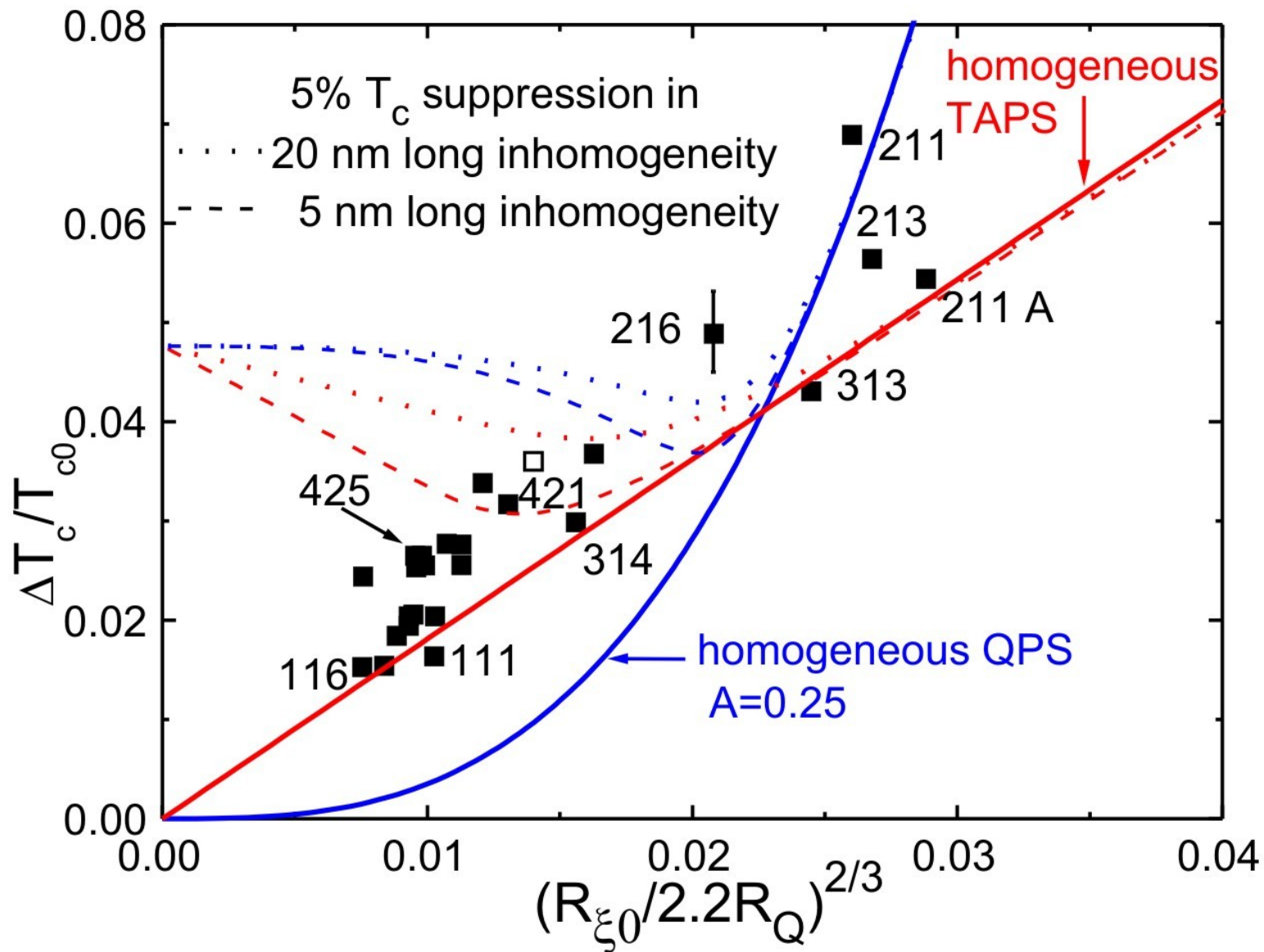
We studied more than 40 samples: 4 and 8 nm thick nanowires, width from 30 nm to 60 nm, length 5 μm . R_N varied from 15 Ω to 100 Ω .



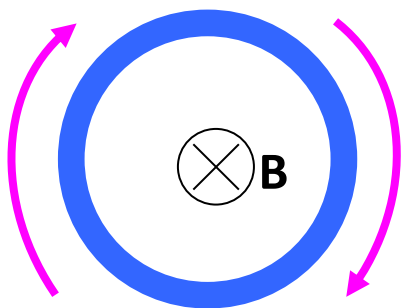
The experimental width of the $R(T)$ transition correlates well with the resistance in normal state R_N . The shape of the majority of $R(T)$ transitions can be nicely fit with TAPS model, while for the thinnest nanowires the best fit is provided with QPS.

K. Yu. A., Ramos-Álvarez, A. V. Semenov, Yu. P. Korneeva, P.P. An, A. A. Korneev, A. Murphy, A. Bezryadin, and G. N. Gol'tsman, **Nanotechnology** 27, 47LT02 (2016)

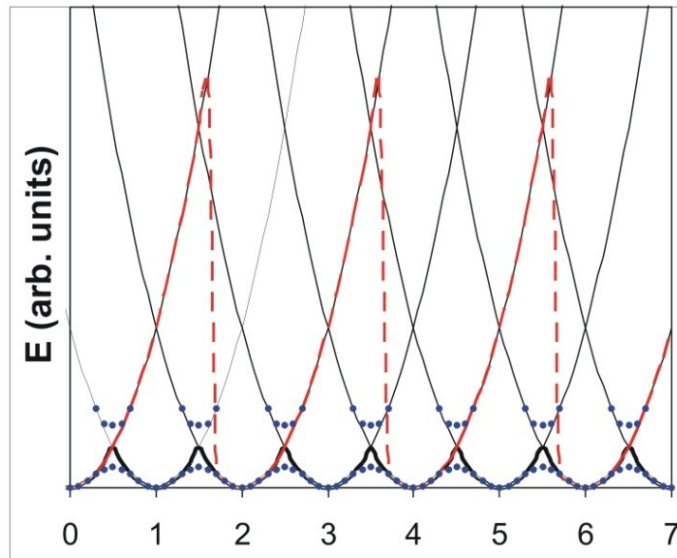
TAPS vs. QPS fits: role of inhomogeneities



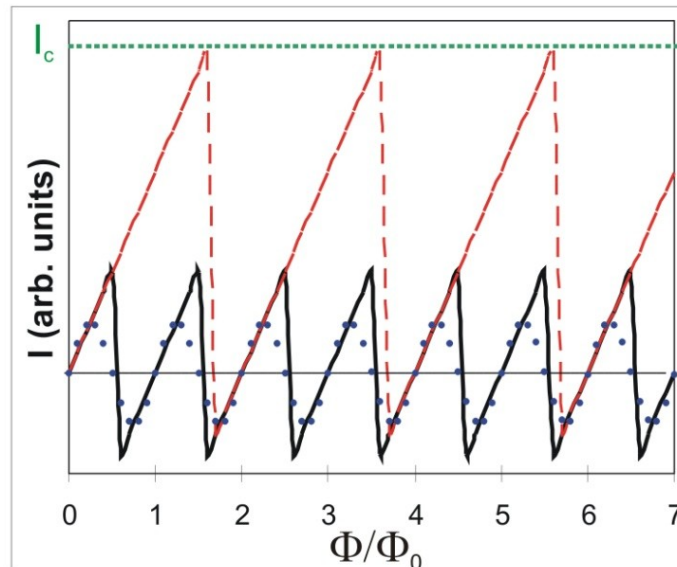
Persistent currents in nanorings



Persistent currents in a very narrow superconducting loop governed by quantum fluctuations



Quantum phase slips open gap in the energy spectrum of a very narrow superconducting loop: coherent superposition of $+2\pi$ and -2π phase slips.
K. A. Matveev, A. I. Larkin, L. I. Glazman, PRL 89, 096802 (2002)
K.A., D. S. Golubev, A.D Zaikin, Phys. Rep. 464, 1 (2008)



For a loop of large and fixed circumference the reduction of the line width leads to QPS-governed regime which should result in:

- 1) Reduction of the magnitude of oscillations
- 2) Instead of saw-tooth → sine-type shape

+

- 3) Reduction of the period of oscillations

Oscillations of the persistent current → oscillations of the tunnel current

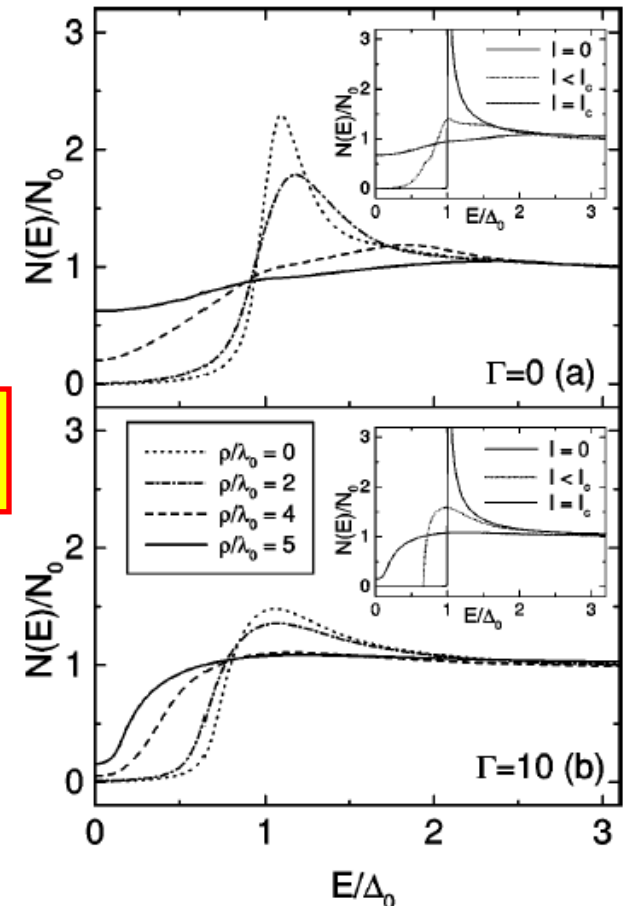
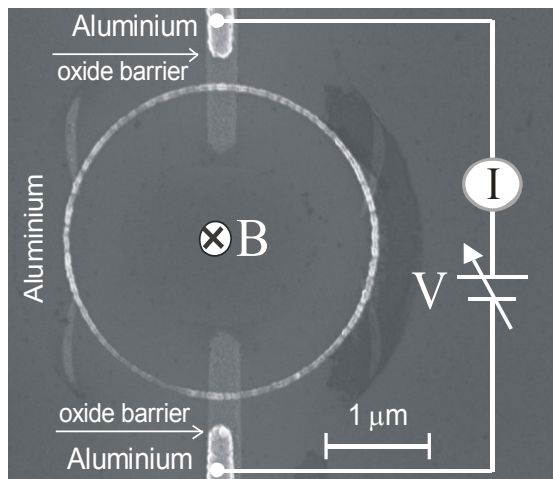
Experiment measuring DOS should probe the supercurrent I

Tunnel current of NIS junction is:

$$I_T = \frac{1}{eR_T} \int_{-\infty}^{\infty} \frac{N_s(E, \Delta[\Gamma(I)])}{N(0)} [f(E) - f(E + eV)] dE$$

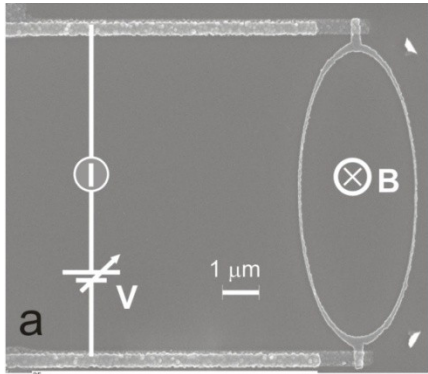
where $N_s(E, \Delta[\Gamma(I)])$ is non-equilibrium DOS

NIS (or SIS) tunnel experiment on a superconducting loop in magnetic field should probe the circulating persistent current

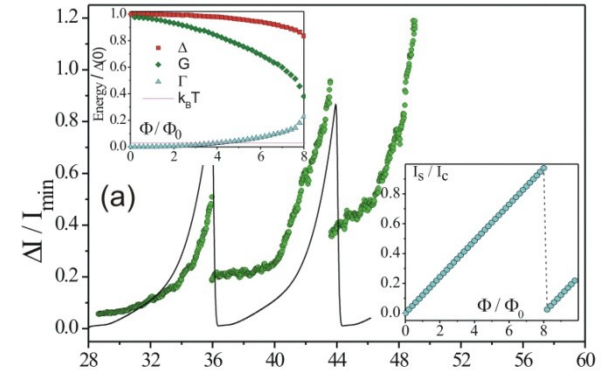


J. Sanchez-Canizares, J. Ferrer and F. Sols,
PRB 63, 134504 (2001)

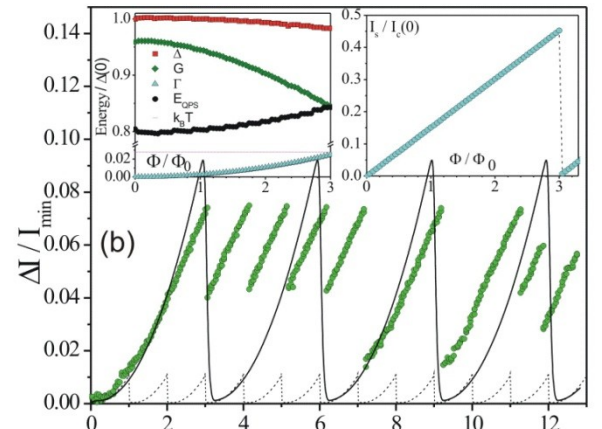
Amplitude and period of oscillations as function of the loop linewidth (aluminium)



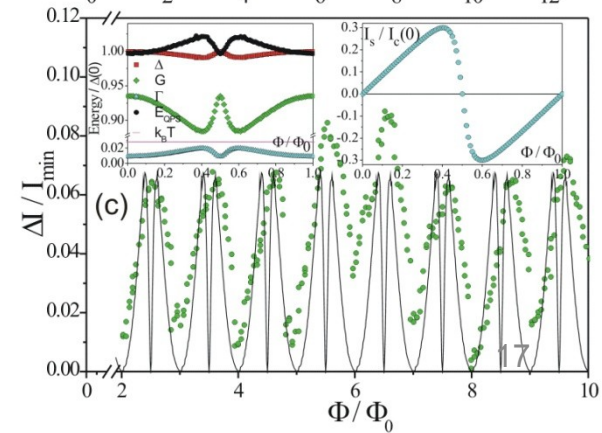
Wide wire: classic limit
 110 nm x 75 nm wire,
 $T_{bath}=65\pm 5$ mK, $\sigma_{fit}^{1/2}=90.8$ nm



Intermediate regime:
 $T_{bath}=52\pm 5$ mK. Solid line:
 $\sigma_{fit}^{1/2}=12.49$ nm and $\Delta\Phi/\Phi_0=3$
 period, dashed line - calculated
 $\Delta\Phi/\Phi_0=1$ oscillations in a slightly
 narrower loop $\sigma_{fit}^{1/2}=12.37$ nm



QPS limit:
 same sample as in (b) further
 gently sputtered at $T_{bath}=54\pm 5$ mK
 Solid line - calculations in the QPS
 limit with $\sigma_{fit}^{1/2}=12.15$ nm



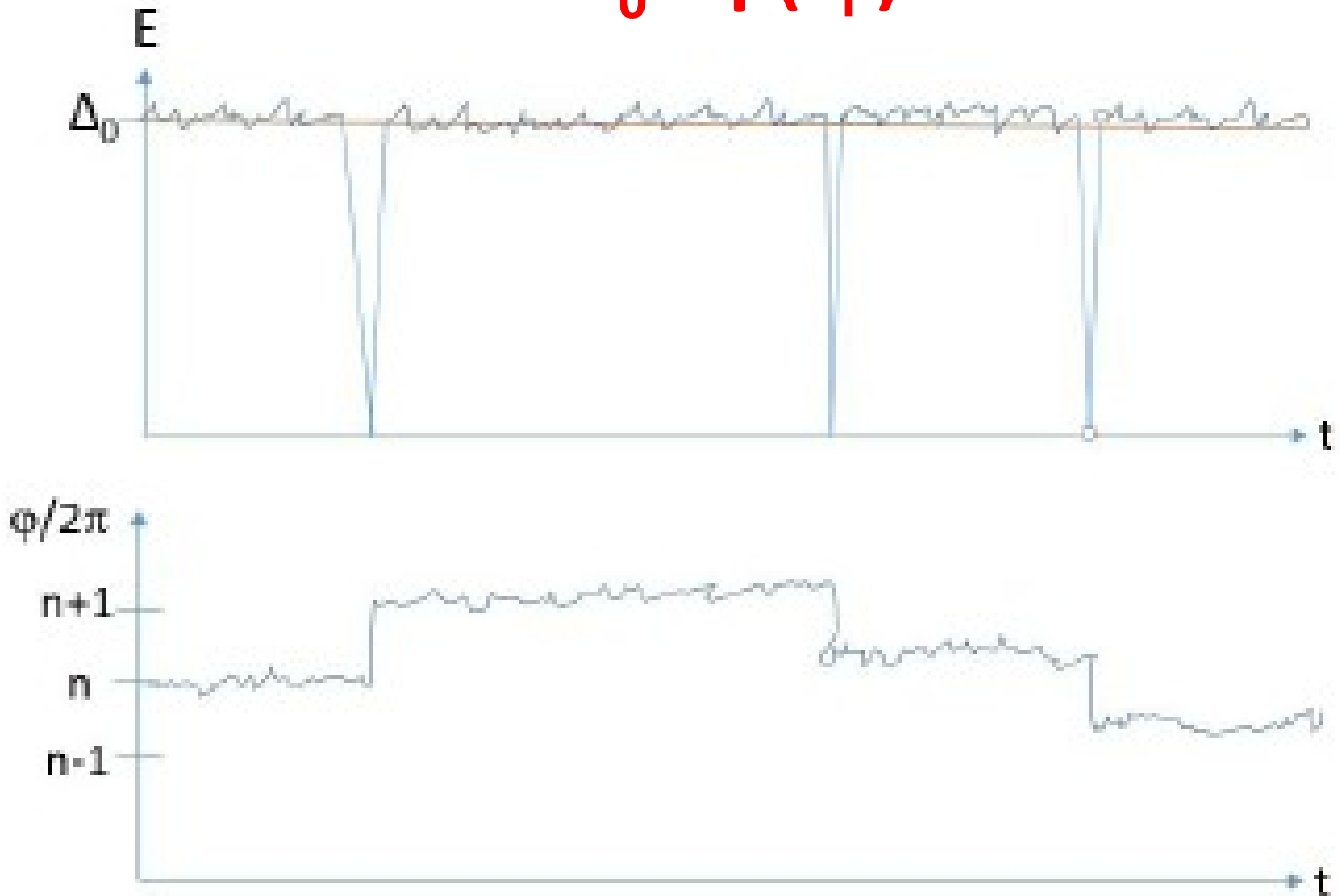
Conclusions on quantum phase slips

Quantum fluctuations of phase in narrow superconducting nanowires suppress all 'basic' attributes of superconductivity manifesting as:

- Broadening of the $R(T)$ transition. In thinnest samples zero resistance is not reached even at $T \rightarrow 0$.
- Suppression of persistent currents in nanorings.

**Fluctuations of the
superconducting gap
modulus**

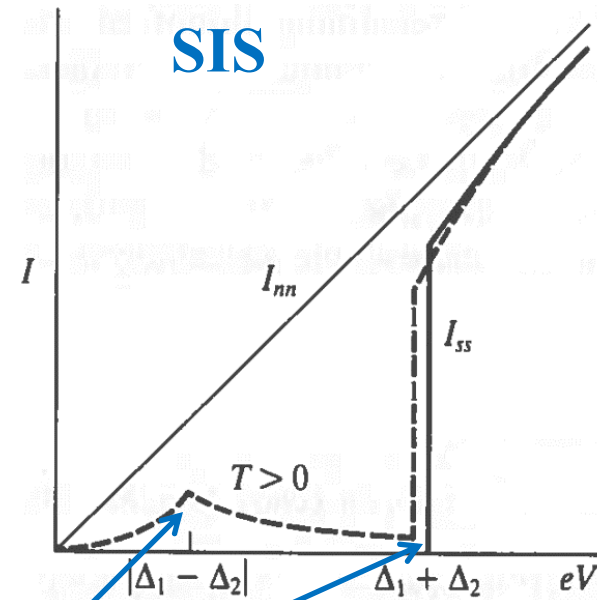
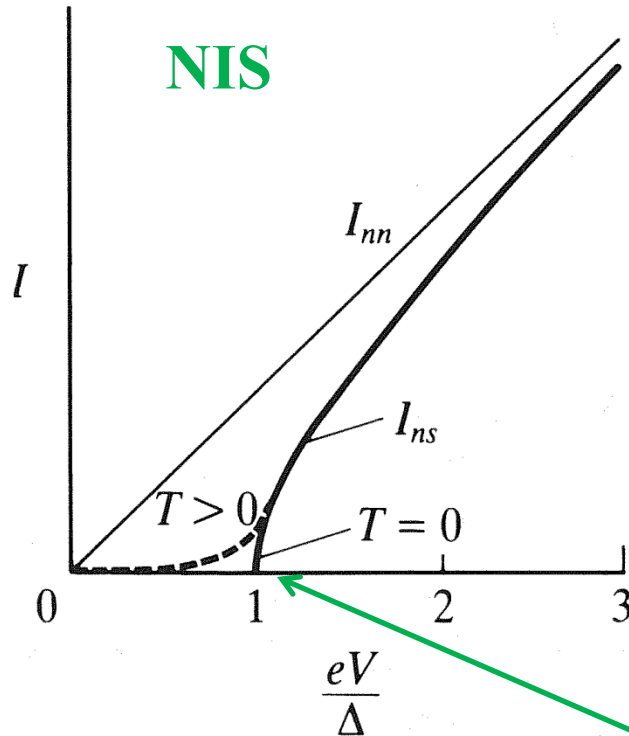
Fluctuations of the order parameter

$$\Delta = \Delta_0 \exp(i\varphi)$$


Tunneling in NIS and SIS systems

Quantum fluctuations of Δ modulus should lead to extra (size dependent) broadening of the gap edge, which should affect the shape of I-V characteristic.

M. Tinkham, Introduction to superconductivity, Mc. Graw-Hill, 1996



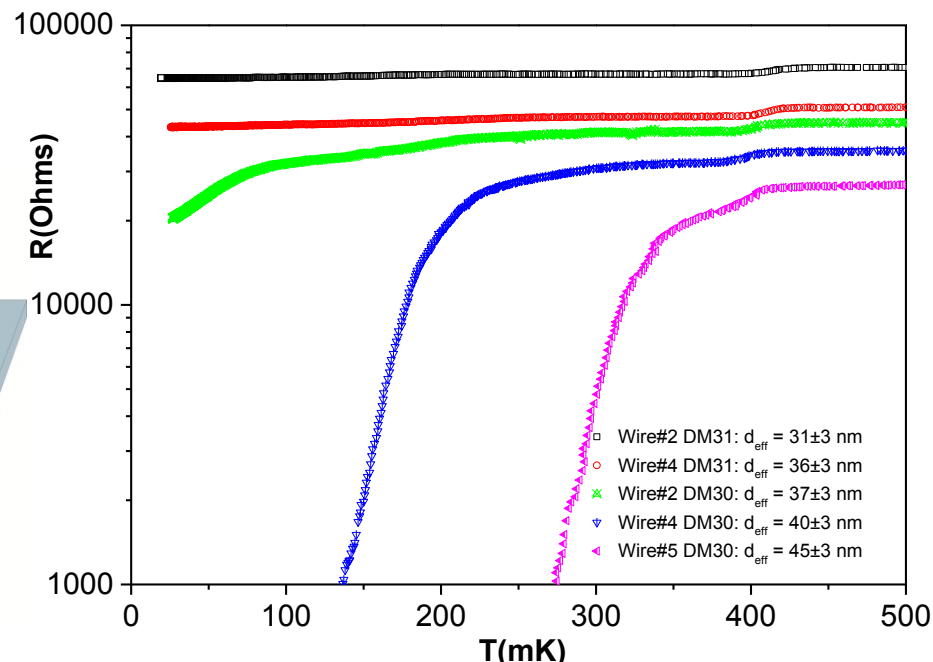
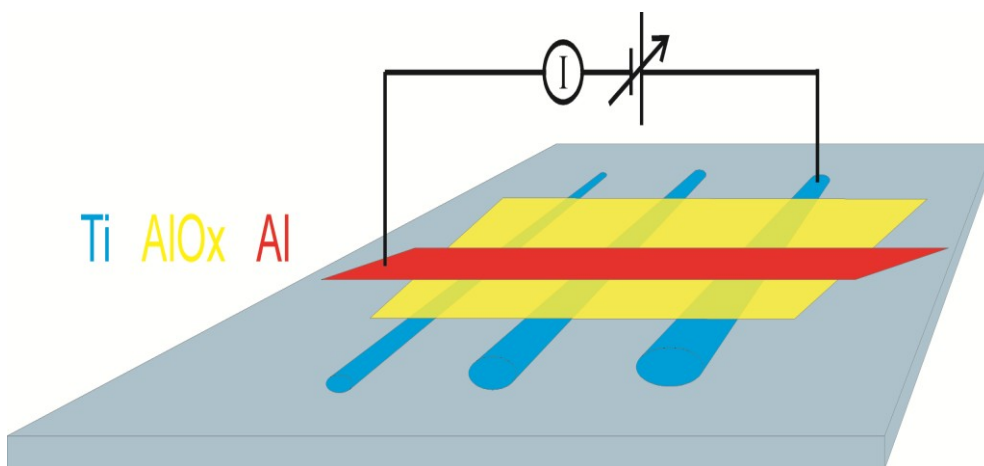
Note: for SIS junction even at finite temperature gap edge singularity is infinitely sharp!

Typically finite broadening of DOS is characterized by phenomenologic Dynes parameter Γ :

$$N(E) \approx N(E_F) \operatorname{Re} \left(\frac{E + i\Gamma}{\sqrt{(E + i\Gamma)^2 - \Delta^2}} \right)$$

Experiment

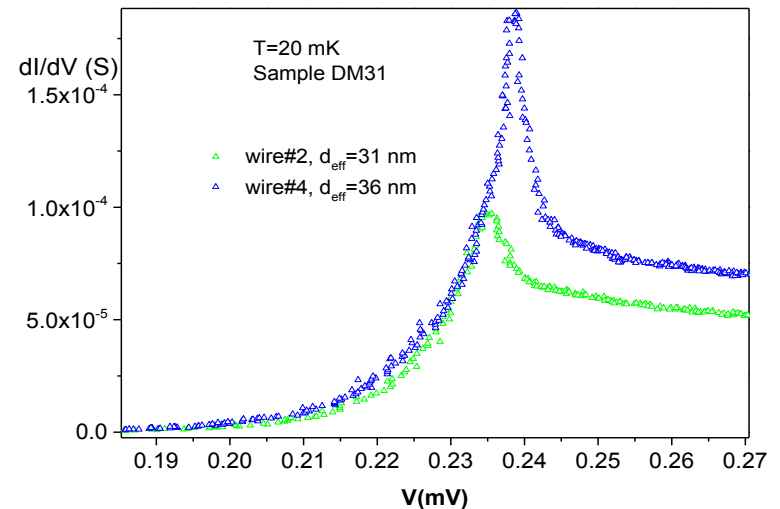
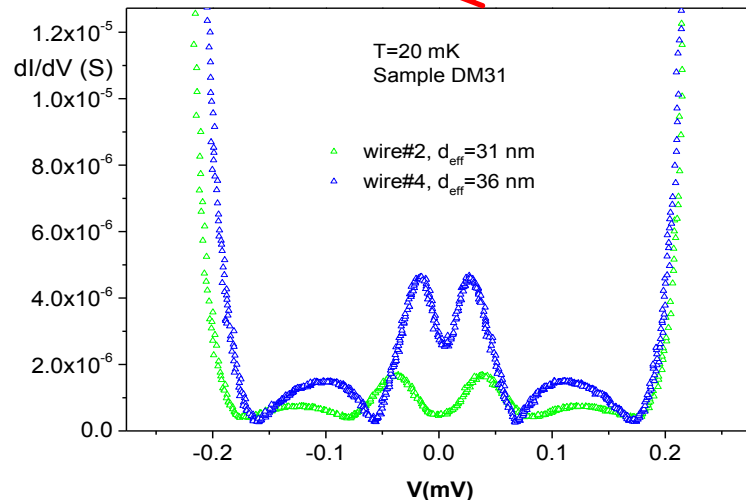
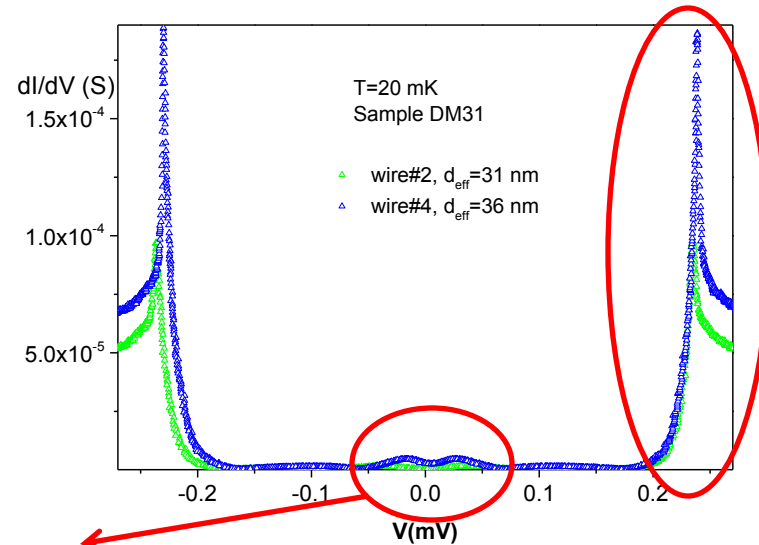
Schematic of the experiment



$R(T)$ of each nanowire and $I(V)$ characteristic between the nanowire and the aluminium electrode can be measured independently.

$R(T)$ dependencies at these diameters are broad associated with QPS.

SIS I(V) at various diameters of nanowires



$\Delta_1 - \Delta_2$ features and

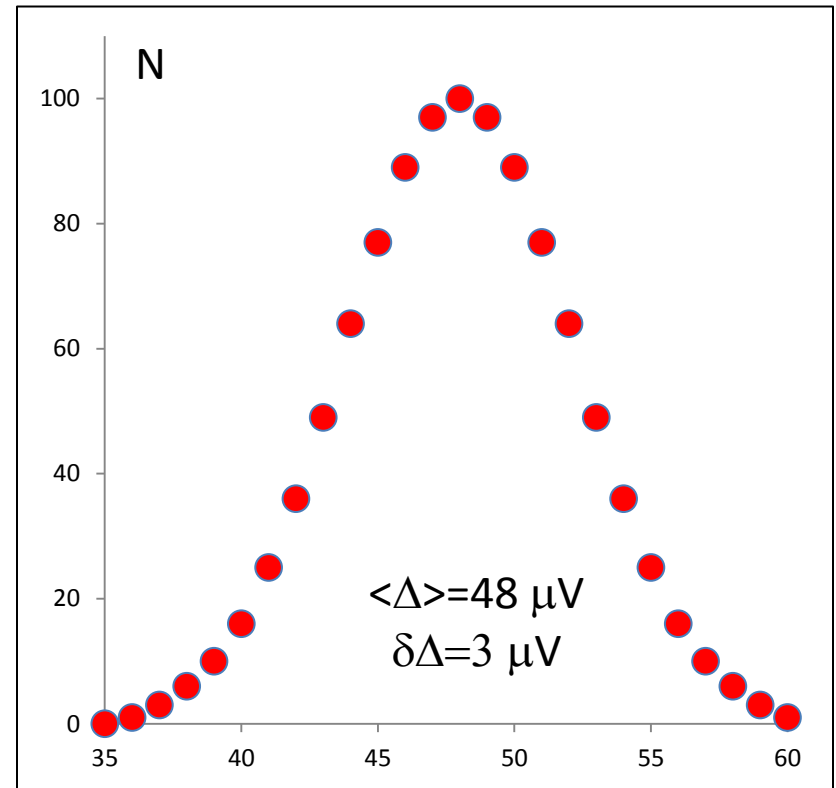
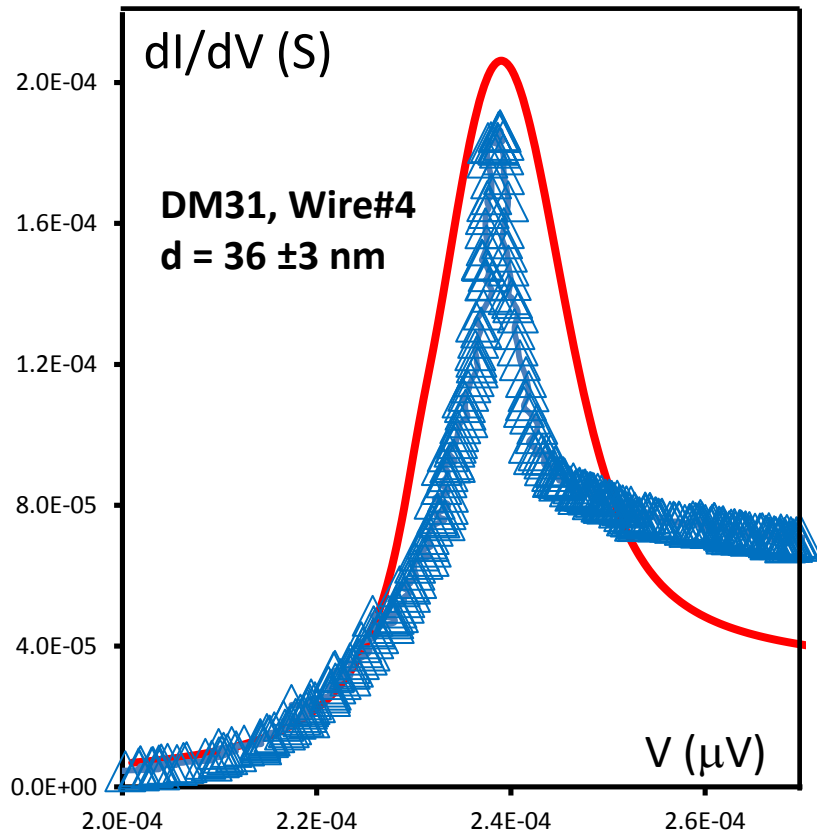
Coulomb blockade and/or Josephson current (?)

$\Delta_1 + \Delta_2$ features

The smaller the nanowire diameter (1) the smaller the average value $\langle \Delta \rangle$ and (2) the larger the gap smearing $\delta \Delta$.

Simulations

We simulate $I(V)$ dependencies assuming Gaussian distribution of the gap fluctuations centered at $\langle\Delta\rangle$ with standard deviation $\delta\Delta$.



Limitations & postulates:

- Gaussian distribution of quantum fluctuation amplitude
 - Cut-off $\Delta_{\text{bulk}}/2 < \Delta < \Delta_{\text{bulk}}$
- Finite magnitude of voltage modulation and finite lock-in integration time

Other contributions

In 1-D superconductors QPSs, related to fluctuations of the phase φ , can excite 'soft' plasmonic mode (Mooij-Schoen mode). Electrons interact with these plasmonic excitations which cause re-distribution of electronic states. The process leads to broadening of DOS singularity at the gap edge Δ . At low temperatures $k_B T \ll \Delta$ such DOS singularity can be formally described by a parameter similar to the phenomenologic Dynes parameter Γ :

$$N_\varphi(E) = N_0(E_F) \operatorname{Re} \left\{ \frac{E + i\Gamma}{\sqrt{(E + i\Gamma)^2 - \Delta^2}} \right\} \sim \exp \left\{ \frac{E - \Delta}{T} \right\}$$

, where $\alpha = \frac{e^2}{2\pi C_l v}$ – parameter, s - wire cross section,

C_l – wire capacitance per unit length, $v = \sqrt{\frac{\pi \Delta s \sigma_N}{C_l}}$ -

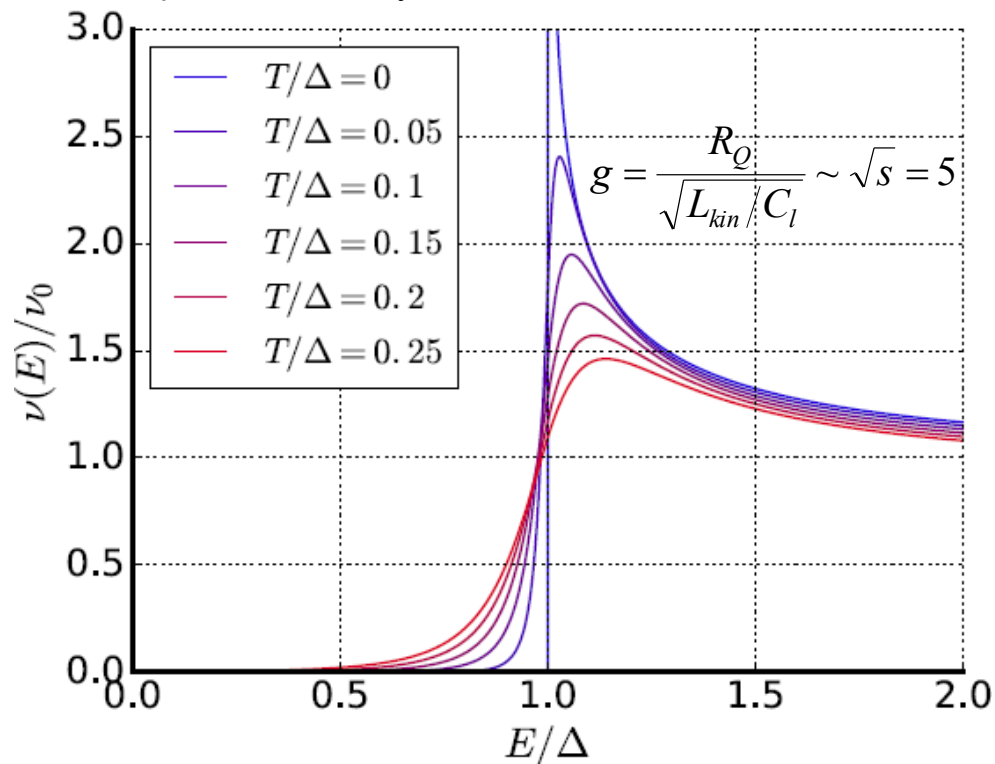
plasmon velocity

DOS broadening depends on:
(1) nanowire cross section s ,
(2) normal state resistivity σ_N ,
(3) temperature

$$\Gamma = (\alpha T)^{1-\alpha}, \quad \alpha \ll 1$$

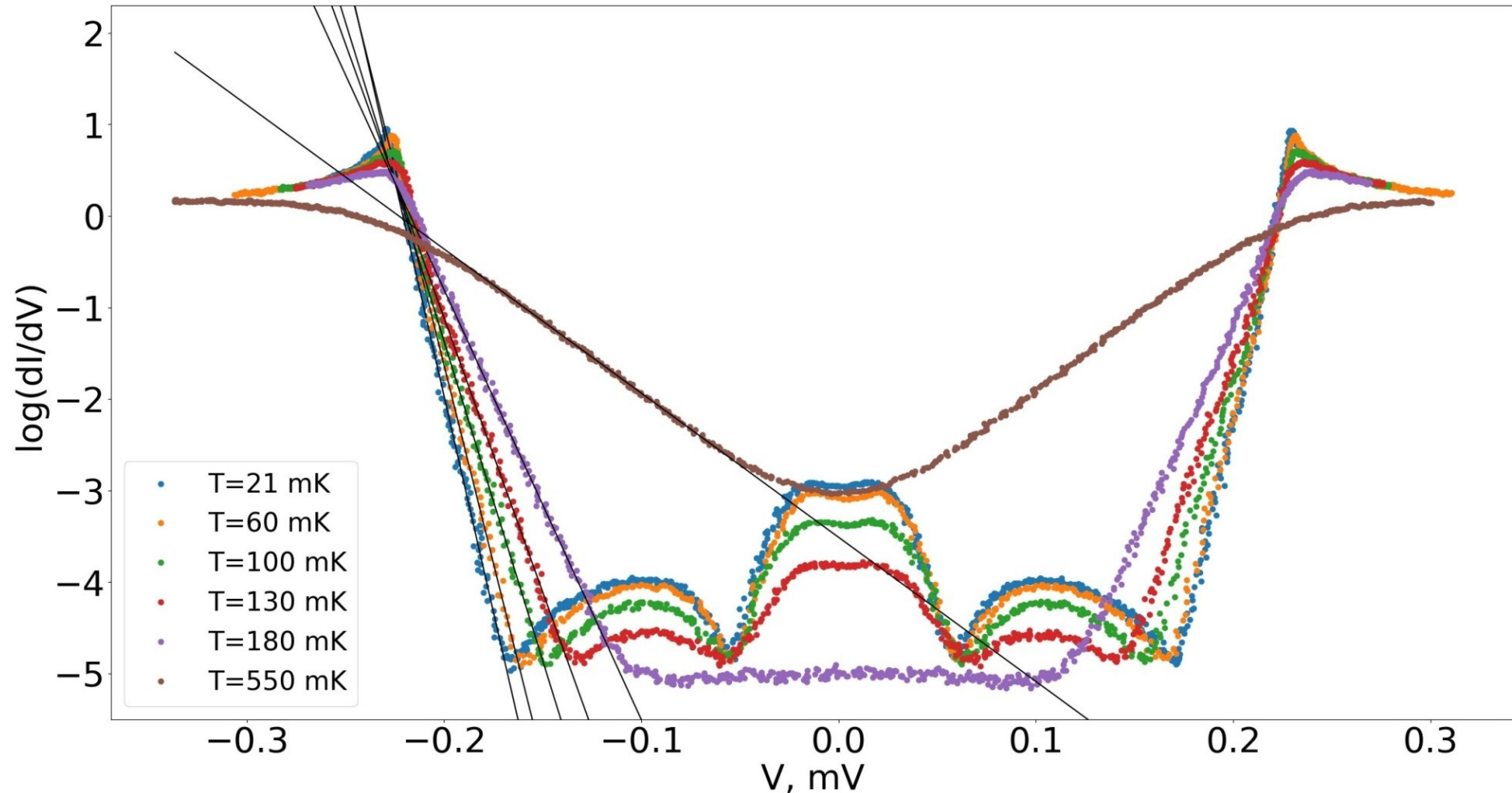
The model does NOT consider:

- fluctuations of the modulus $|\Delta|$;
- relaxation of Mooij-Schoen plasmons;
 - finite length of the nanowire;
- dependence of Δ on cross section s .



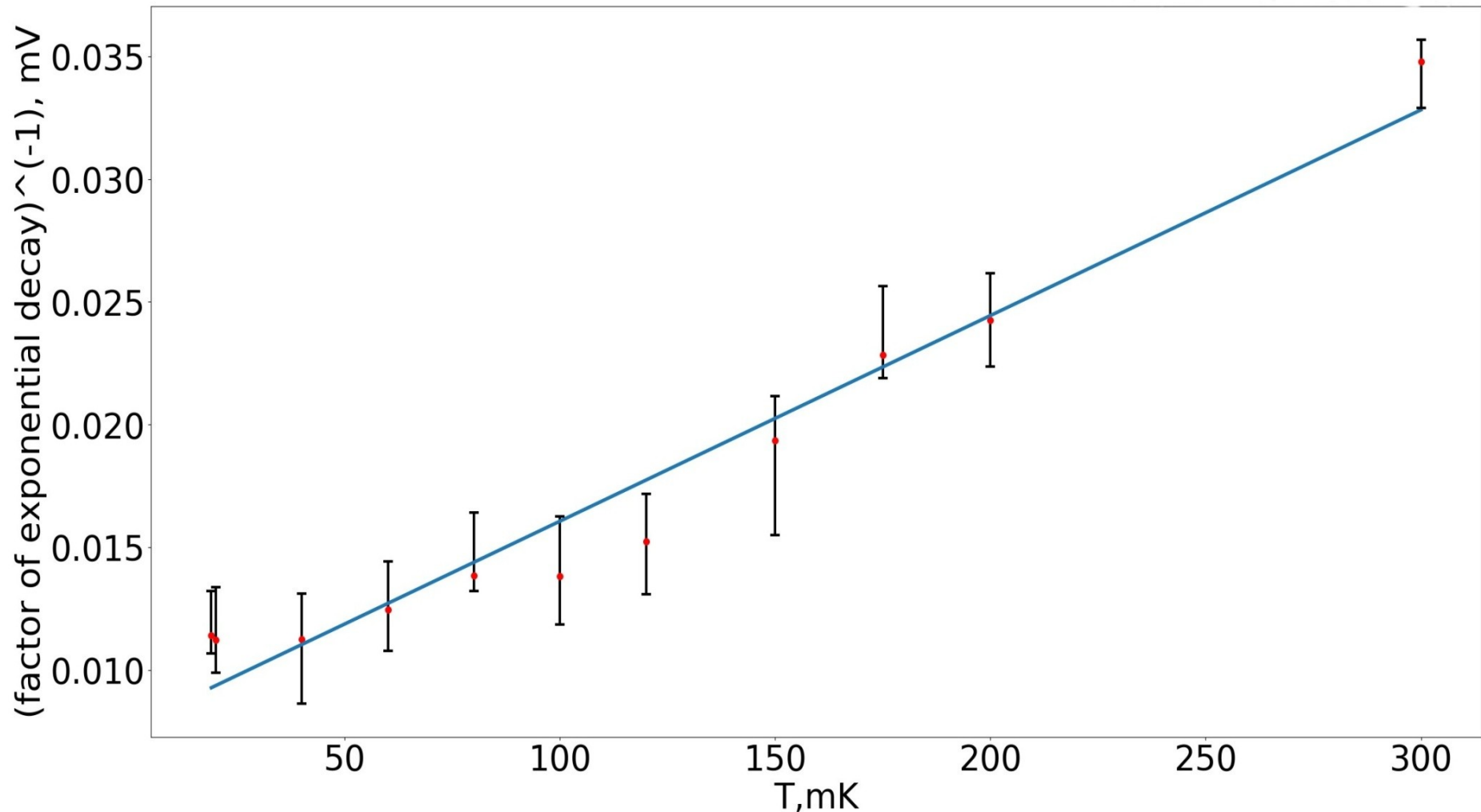
$I(V)$ dependencies

In experiment we can measure both $I(V)$ and dI/dV dependencies. According to the model, DOS (and hence the dI/dV) should exponentially 'leak' below the gap.



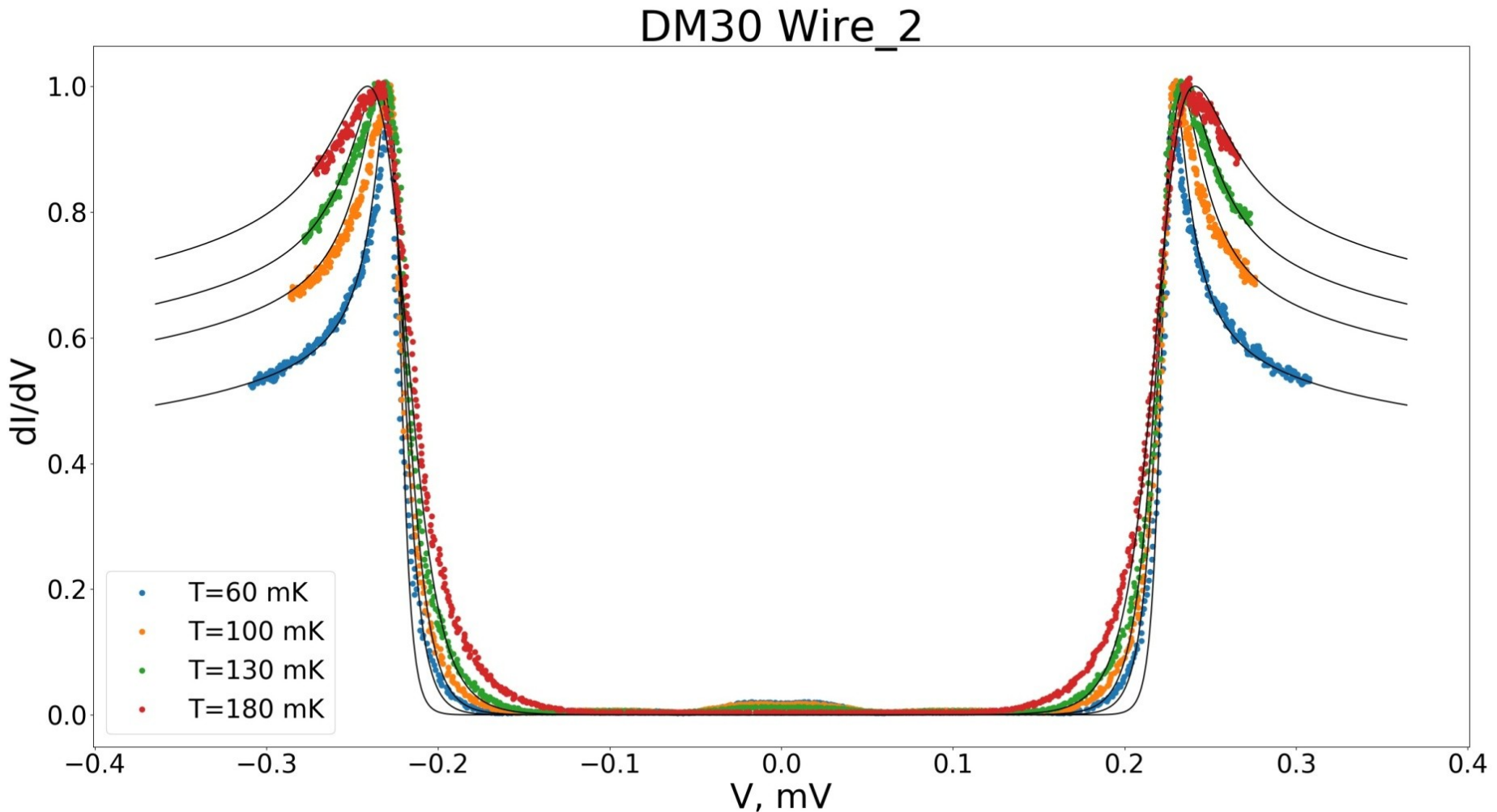
Theory predicts that under the exponent it should be $\sim(E-\Delta/T)$: DOS subgap penetration depends on temperature.

Temperature dependence of DOS decay



At not-too-low temperatures experimentally measured DOS decay is about $8.4 \cdot 10^{-5}$ V/K, which is in a reasonable agreement with theory prediction $k_B/e \approx 8.6 \cdot 10^{-5}$ V/K.

Simulated dI/dV



Experimental dI/dV dependencies are fit with theory using parameter $\alpha=0.585$, being normalized at maximum to 1. The agreement is acceptable, but not perfect. Presumably at such high constant of interaction $\alpha=0.585$, one should take into consideration other contributions.

Conclusions on physics

Quantum fluctuations in narrow superconducting nanowires suppress all 'basic' attributes of superconductivity manifesting as:

- Broadening of $R(T)$ transition. In thinnest samples zero resistance is not reached even at $T \rightarrow 0$.
- Suppression of persistent currents in nanorings.
- Smearing of DOS at energies $\sim \Delta$.

Open questions

- Mechanism of energy dissipation?
- How to separate smearing of DOS at energies $\sim \Delta$ due to phase fluctuations, modulus fluctuations and the Mooij-Schoen plasmons?

Thank you!

Our key publications on QPS

- M. Zgirski, K.-P. Riikonen, V. Touboltsev, and K. Arutyunov. Size Dependent Breakdown of Superconductivity in Ultranarrow Nanowires // **Nano Letters** 5, 1029 (2005).
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