# Quantum Fluctuations in Low-Dimensional Superconductors

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# Outline

## 1. Introduction

Fluctuations & Quantum Phase Slip concept

## 2. Physics

Transport properties:

Broadening of the R(T) transition

Noise

Suppression of persistent currents in nanorings

Smearing of the superconducting gap edge

## 3. Applications

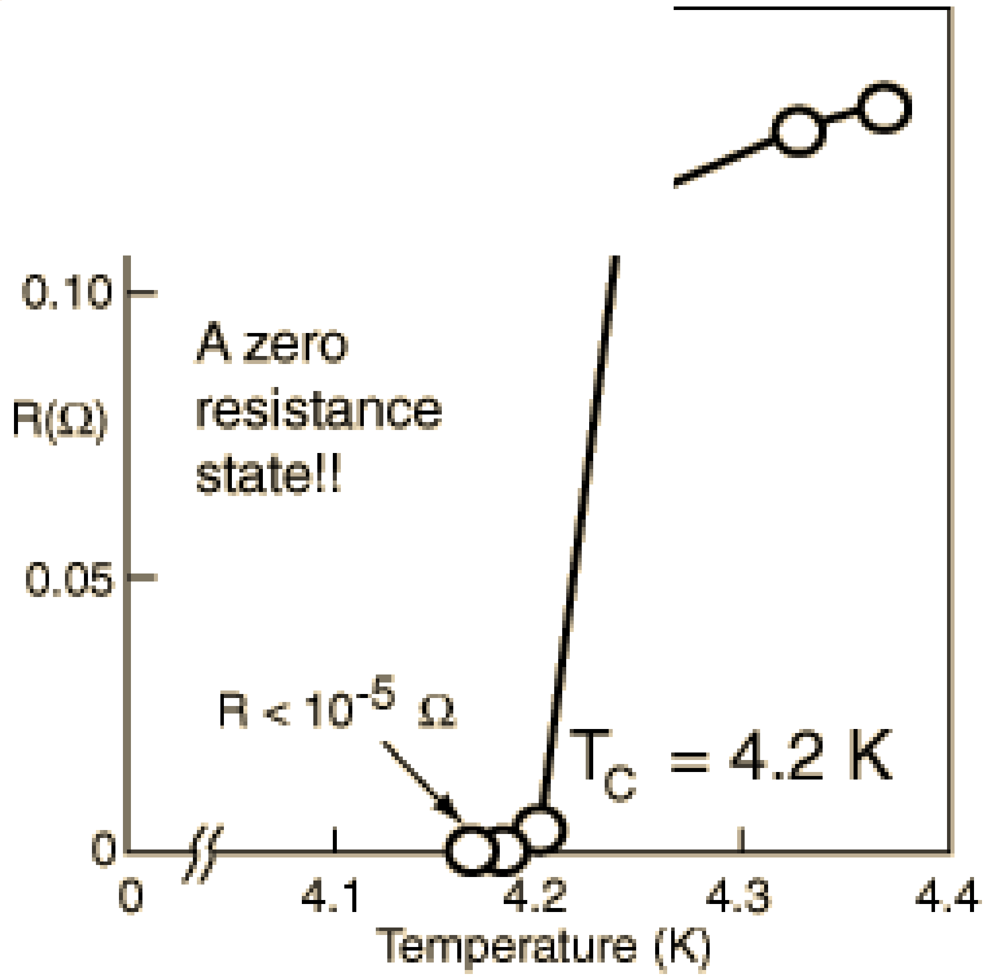
QPS qubits

Junctionless Cooper pair transistor

Quantum standard of electric current

## Conclusions

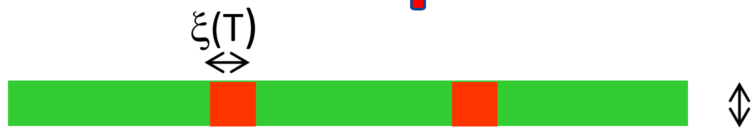
# R(T) transition




H. K. Onnes,
Commun. Phys. Lab.12,120, (1911)

# Fluctuations in a 1D supercondcutor

Long 1D wire of cross section $\sigma$




$\sqrt{\sigma} < \xi(T) << L$

If the wire is infinitely long, there is always a finite probability that in some fragment(s) the magnitude of the order parameter instantly becomes zero and the phase changes by $2\pi$

The minimum length the superconductivity can be destroyed is the coherence length $\xi(T)$

The minimum energy corresponds to destruction of superconductivity in a volume $\xi(T)\,\sigma$:
$\Delta F = B_c^2\,\xi(T)\,\sigma$, where $B_c(T)$ is the critical field

In the limit <u>rare</u> events the probability of the process $P(T) \sim \exp(-\Delta F / \mathcal{E})$

**Thermal activation: $\mathcal{E} \sim k_B T$.**

**Important at T→Tc.**

**Quantum: $\mathcal{E} \sim \Delta$.**
**Weak temperature dependence, exist even at T→0**

In current state the particular manifestation of a quantum fluctuation when magnitude of the order parameter momentary nulls and phase changes by $\pm 2\pi$ is often called "phase slip"

# QPS contribution

Full model (G-Z) →

*A. Zaikin, D. Golubev, A. van Otterlo, and G. T. Zimanyi, PRL **78**, 1552 (1997)*
*A. Zaikin, D. Golubev, A. van Otterlo, and G. T. Zimanyi, Uspexi Fiz. Nauk **168**, 244 (1998)*
*D. Golubev and A. Zaikin, Phys. Rev. **B 64**, 014504 (2001)*

In the limit $R(T) \ll R_N$ QPSs are activated at a rate:
$\Gamma_{QPS} = E_{QPS}/h = \Delta_0[(R_Q L^2)/(R_N \xi^2)] \exp(-S_{QPS})$

$$R(T) = b\frac{\Delta_0(T) S_{QPS}^2 L}{\xi(T)} \exp\{-2S_{QPS}\}$$

where $S_{QPS} = A\cdot[(R_Q/\xi)/(R_N/L)]$, $R_Q = h/4e^2 = 6.45\ k\Omega$, $\Delta$ = energy gap,

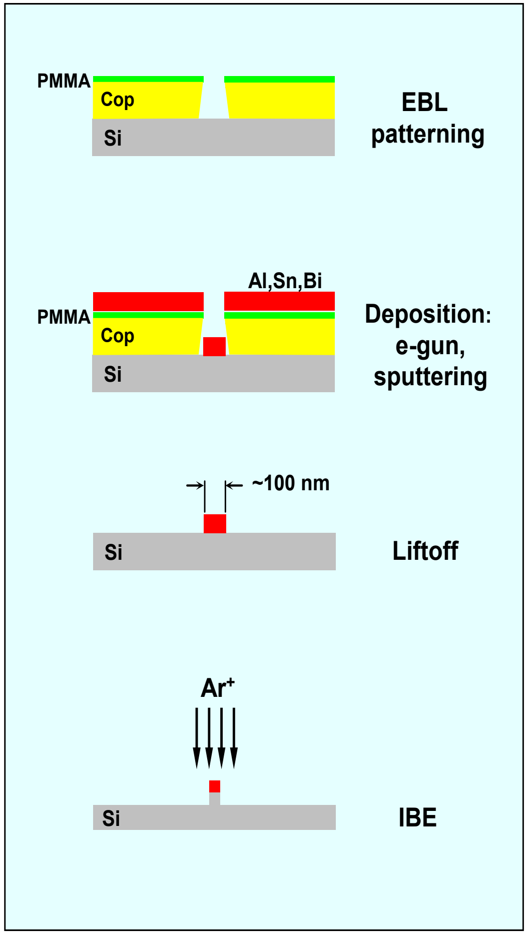
$R_N$ = normal state resistance, L is wire length, $\xi = 0.85(\xi_0 l)^{1/2}$ is coherence length

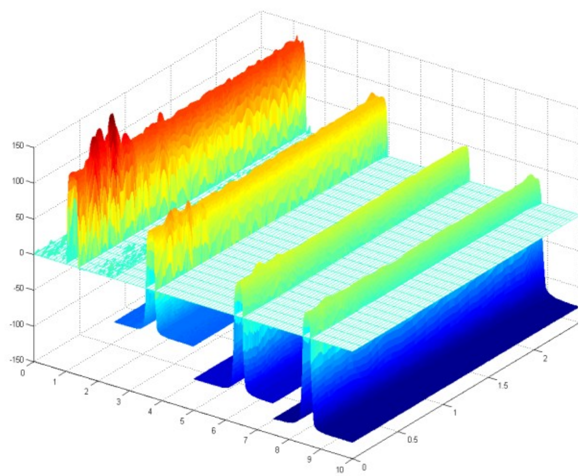
$A \approx b \approx 1$ are numerical parameters.

Essentially, there is only one fitting parameter: $A \approx 1$.
For a dirty limit superconductor: $\Gamma_{QPS} \sim \exp(-A'\sigma T_c^{1/2}/\rho_N)$, where $\sigma$ is the wire diameter and $\rho_N$ is the normal state resitivity.
Materials with low $T_c$ and high resitivity $\rho_N$ are of advantage!

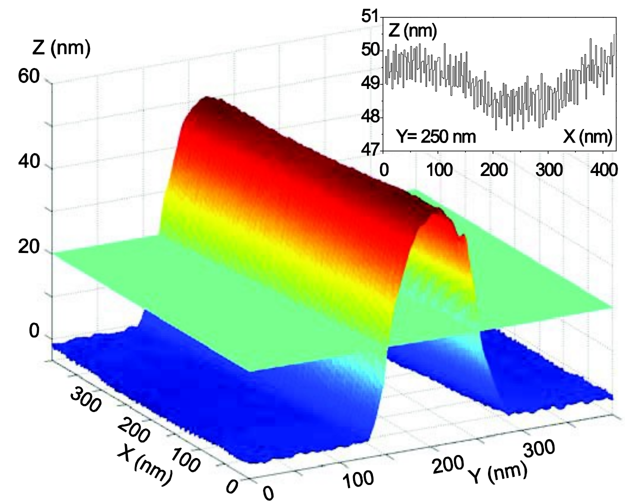
# 1D samples: fabrication & shape control

Objective: to enable measurements of the **same** nanowire with progressively reduced diameter



Evolution of Al nanowire after several sessions of ion milling.

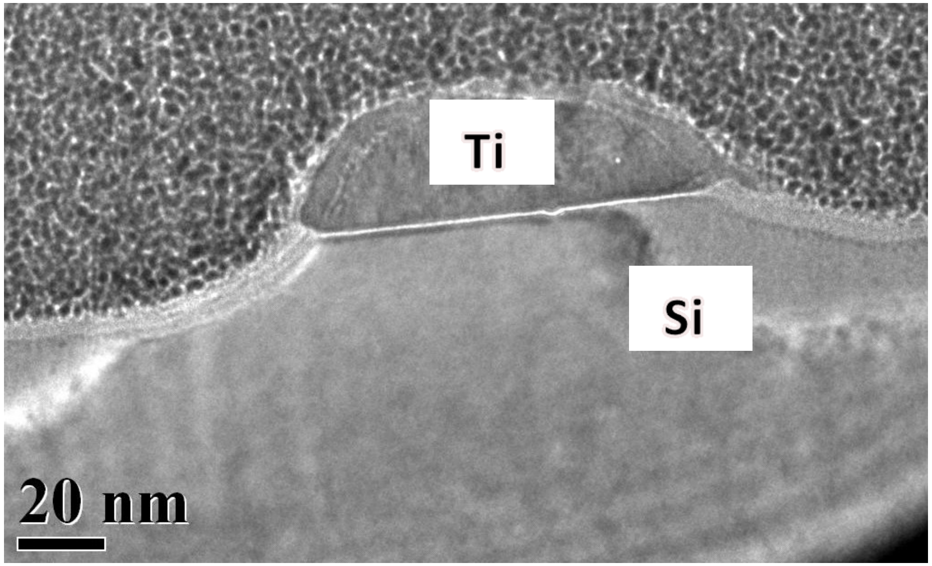


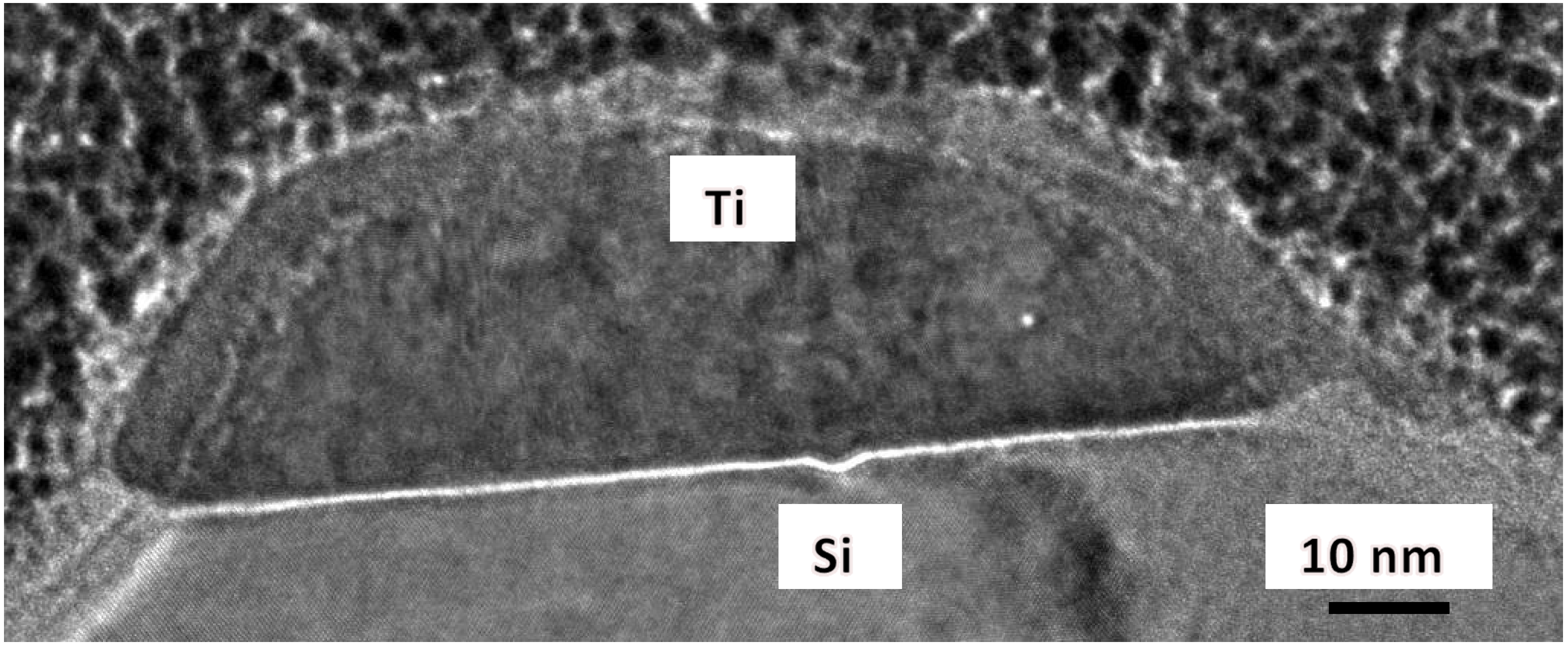

AFM image of a typical Ti nanowire. Inset shows the surface roughness ±1 nm.

**Ion beam provides polishing and gradual reduction of the cross-section**

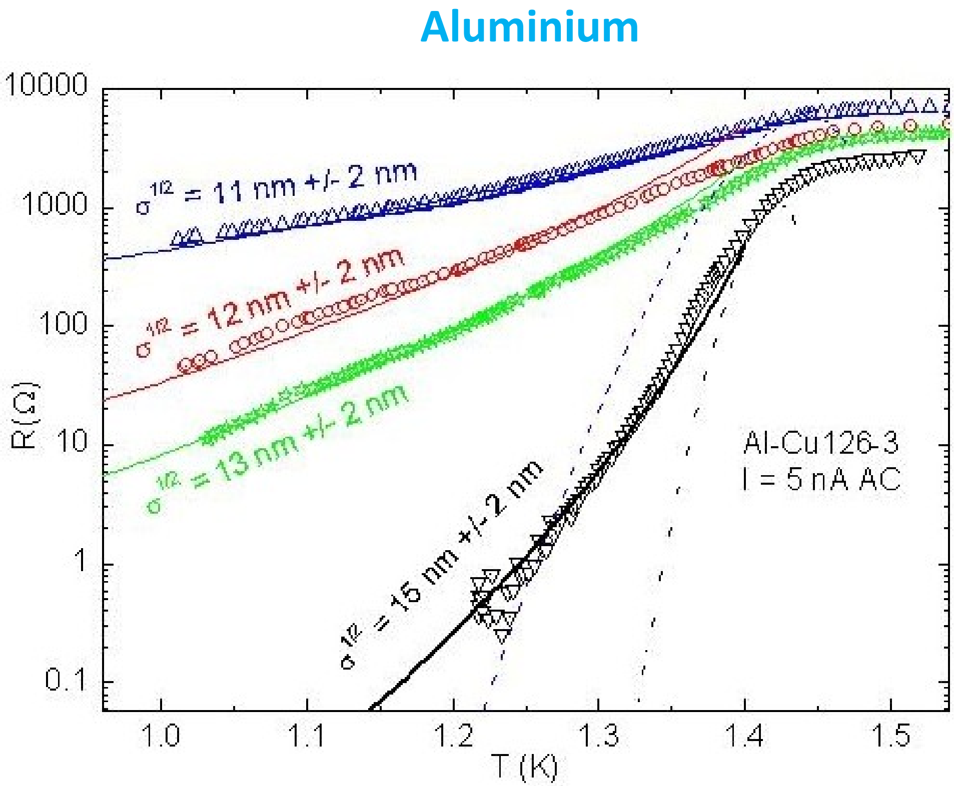
Penetration depth of 1 keV $Ar^+$ ions inside Al or Ti matrix < 2 nm

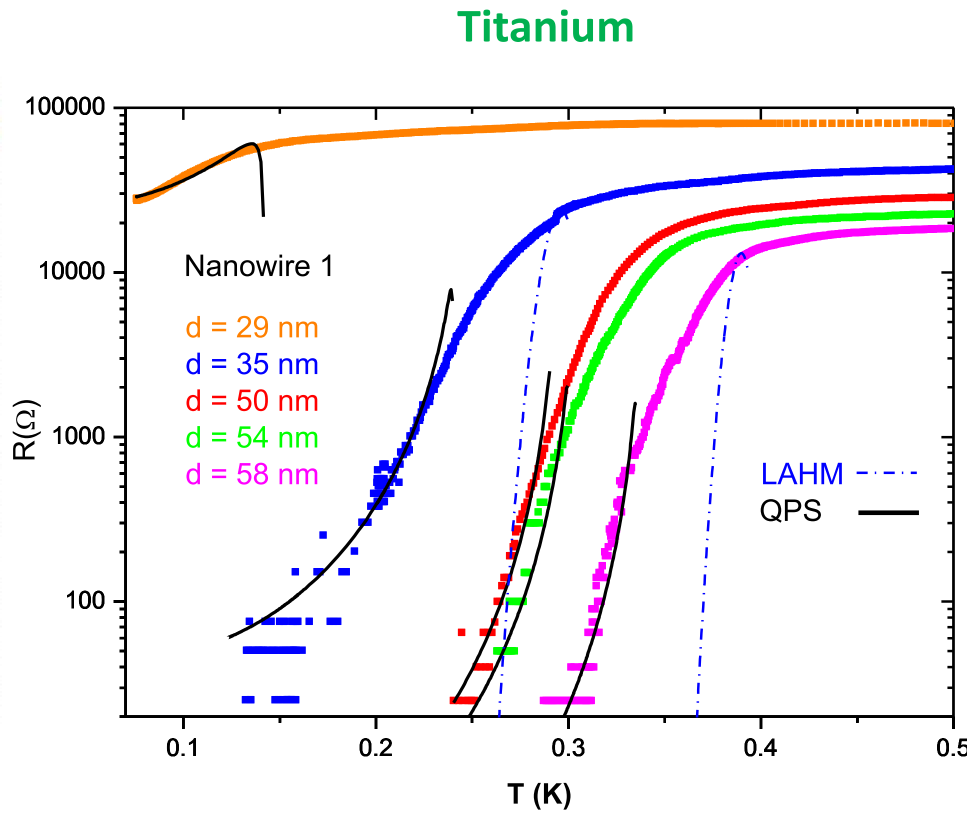
# TEM analysis of the ion milled nanowire cross section



# Manifestation of QPS: broadening of R(T) transition



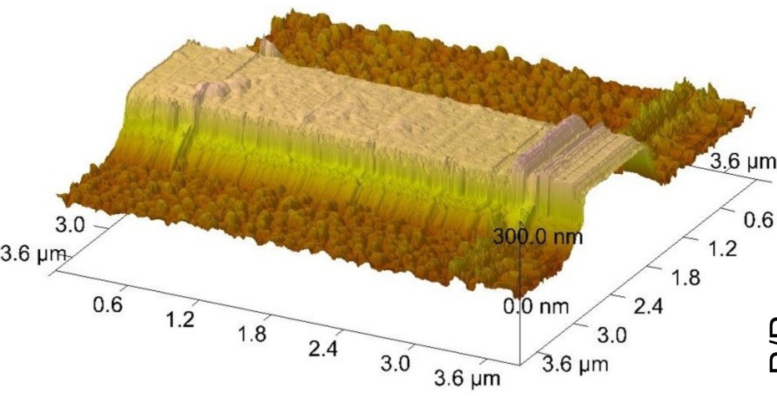

**Dashed lines: thermally activated fluctuations (LAHM) at T ~Tc**

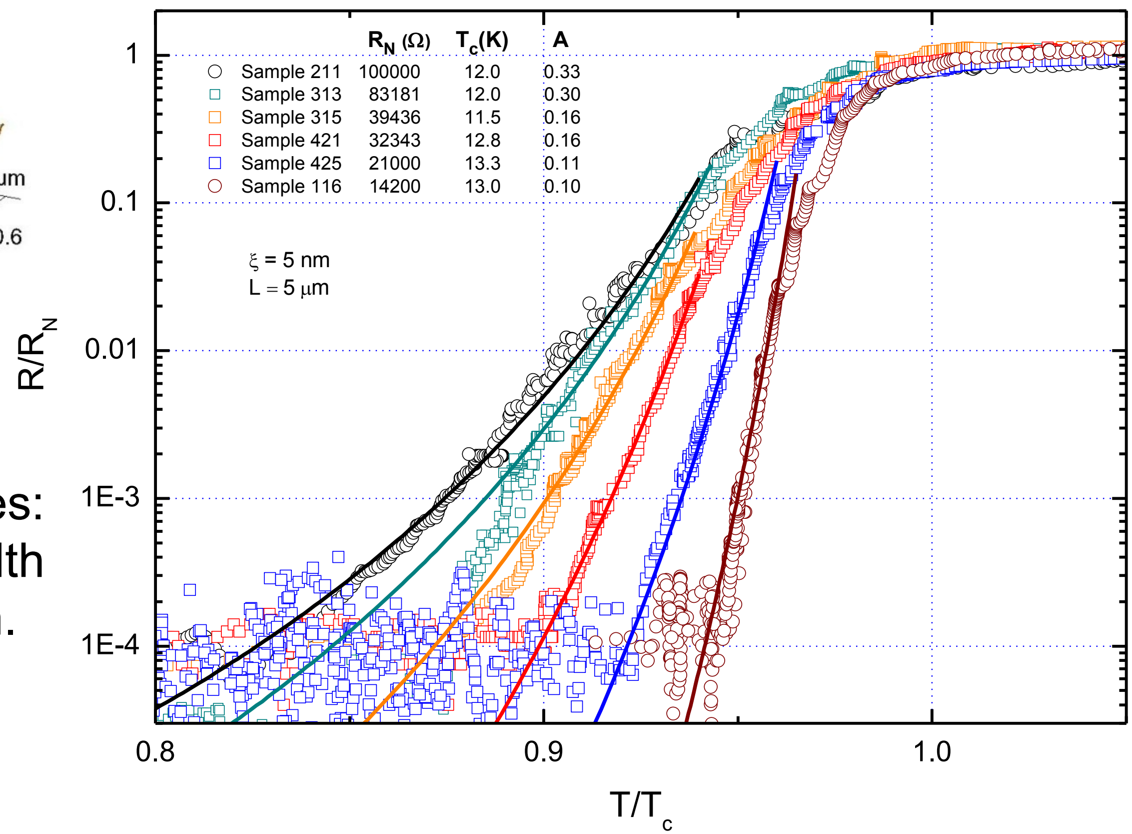
**Solid lines: quantum fluctuations (Golubev – Zaikin) model at T << Tc**

*M. Zgirski, K.-P. Riikonen, V. Touboltsev, and K. Arutyunov. // **Nano Letters** 5, 1029--1033 (2005).*
*M. Zgirski, K.-P. Riikonen, V. Touboltsev and K. Yu. Arutyunov.// **PRB** 77, 054508-1 -- 054508-6 (2008).*
*J. S. Lehtinen, T. Sajavara, K. Yu. Arutyunov, M. Yu. Presnjakov, and A. S. Vasiliev // **PRB** 85, 094508 (2012).*

# NbN nanowires

NbN is a highly disordered (= highly resistive) superconductor




We studied more than 40 samples: 4 and 8 nm thick nanowires, width from 30 nm to 60 nm, length 5 um. $R_\square$ varied from 15 Ω to 100 Ω .

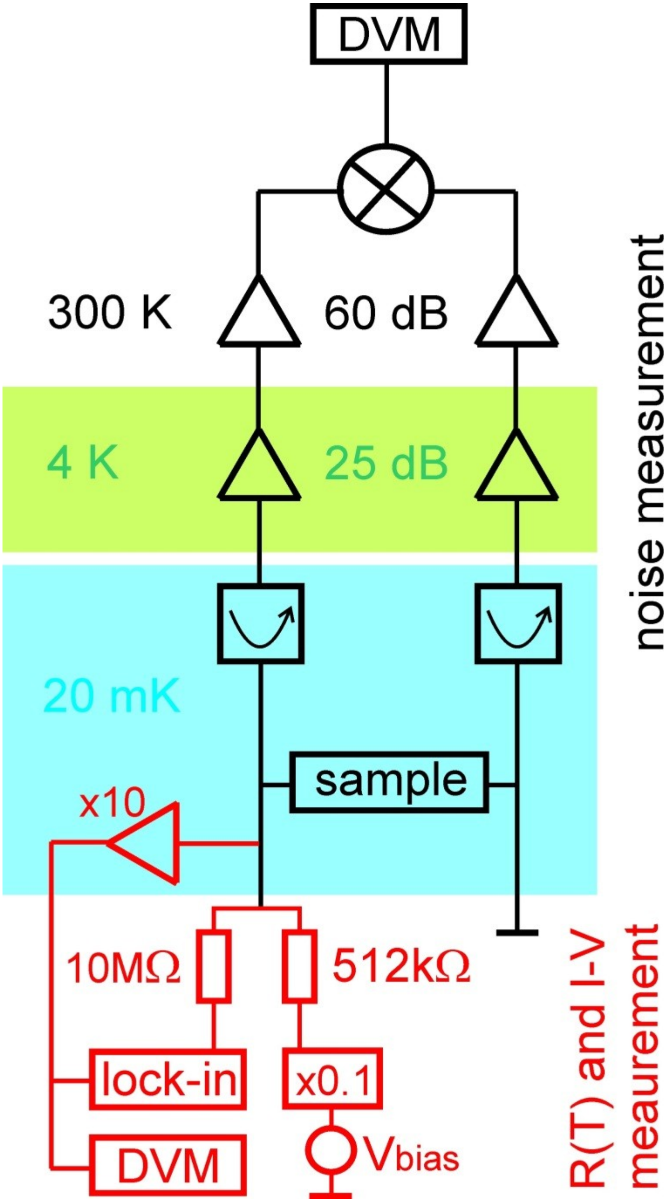
**The experimental width of the R(T) transition correlates well with the resistance in normal state $R_N$. The shape of the R(T) transition can be nicely fit with the model of QPS. As expected, TAPS model provides much steeper transition.**

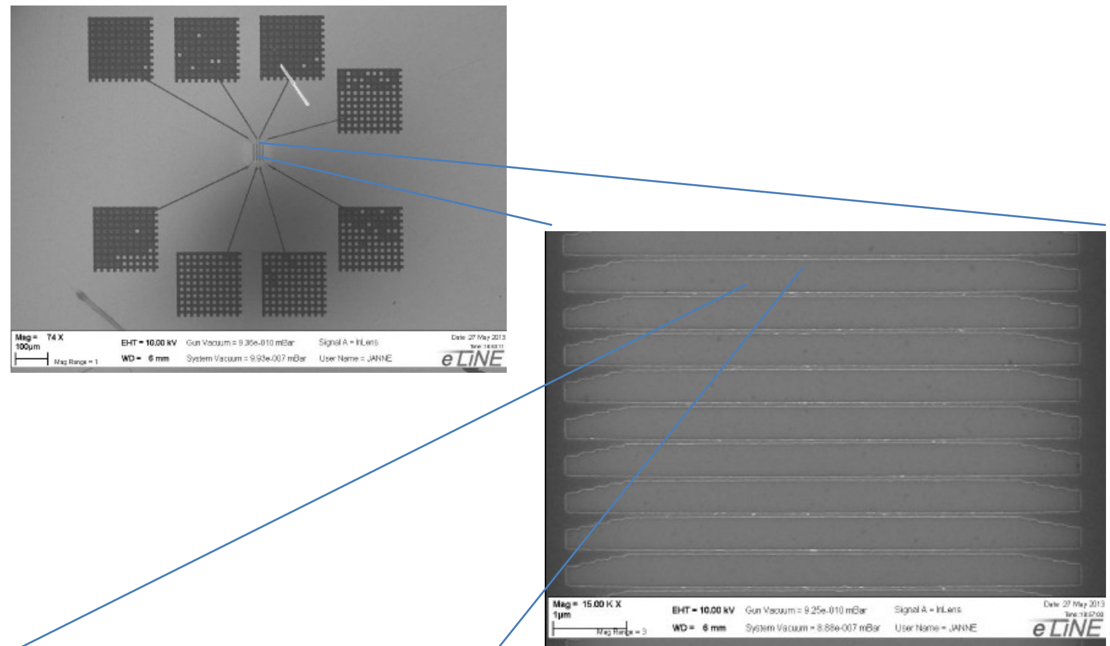
**Samples were fabricated in MPI-Moscow (Goltsman, Korneev, Semenov, An)**

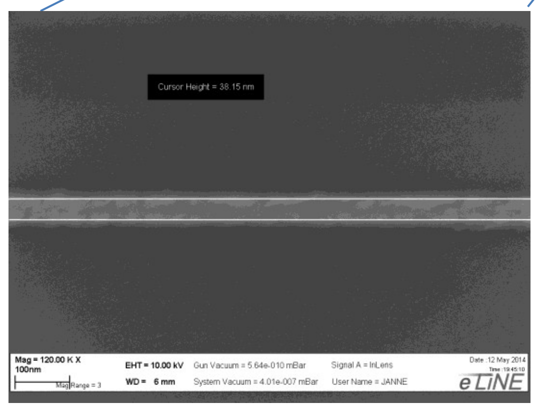
# Noise

In collaboration with
A. Zaikin , A. Semenov
&
P. Lähteenmäki, T. Nieminen and P. Hakonen
*LT Lab, Aalto University, Finland*

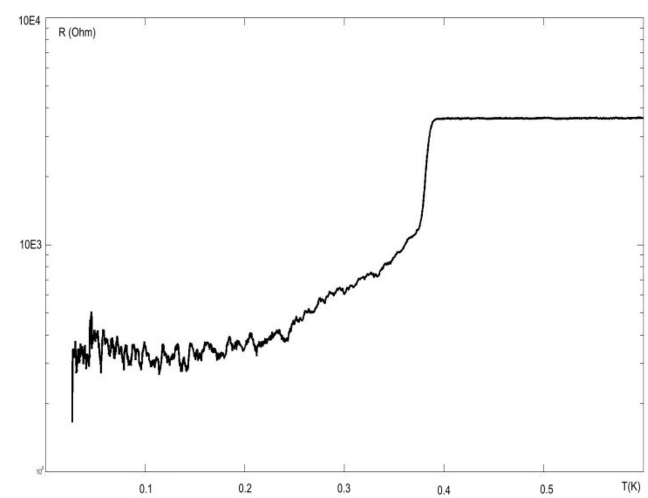
# Experiment



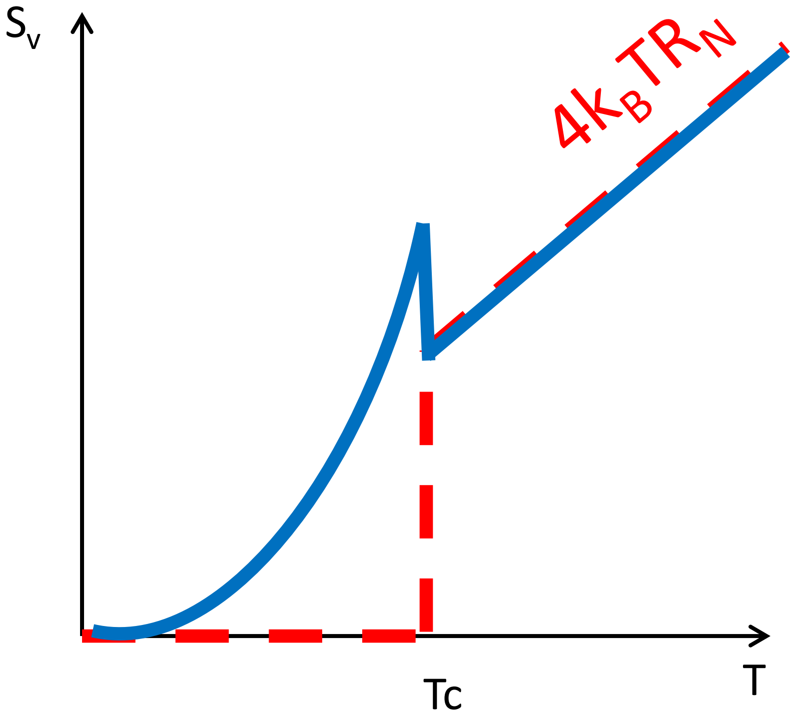



Array of 100 parallel wires



# Expectations

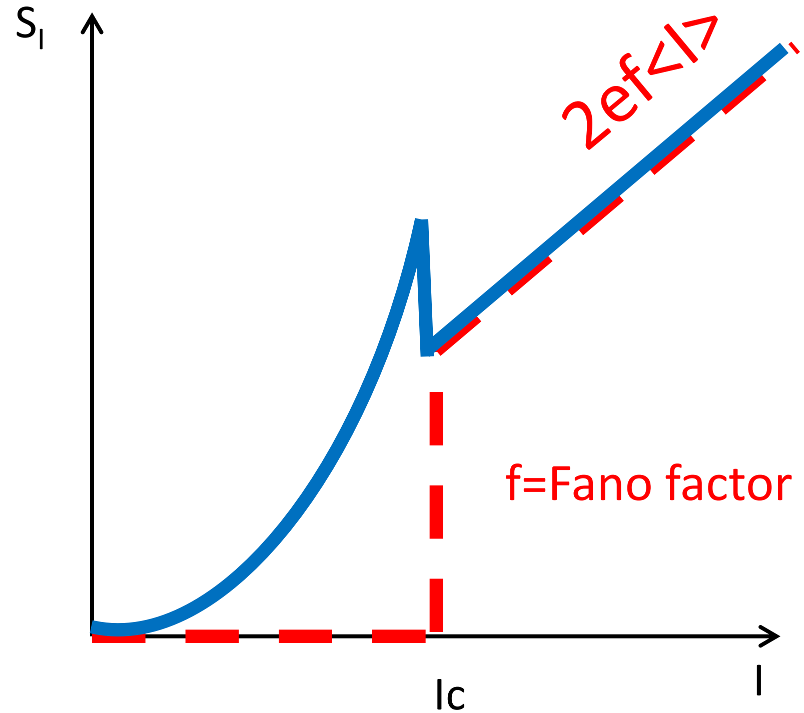
$eV \ll k_B T$ (Johnson noise)




Bulk superconductor

QPS nanowire

$eV \geq k_B T$ (shot noise)




Bulk superconductor

QPS nanowire

# Shot noise due to phase slips

***Current shot noise*** *due to charge pulses:* $Q = \int I dt$

$$S_I = 2eI = 2e^2 f$$ *random pulses – white noise*

***Switch to flux quantum:*** $e \Leftrightarrow \varphi_0$

***Voltage shot noise*** *due to phase slips:* $\Phi = \int V dt$

$$S_V = 2\varphi_0{}^2 f = 2\varphi_0 V$$ *random pulses – white noise*

$$F_V = \frac{S_V}{2\varphi_0 V}$$

$\varphi_0 = \frac{h}{2e}$ ***is the flux quantum***

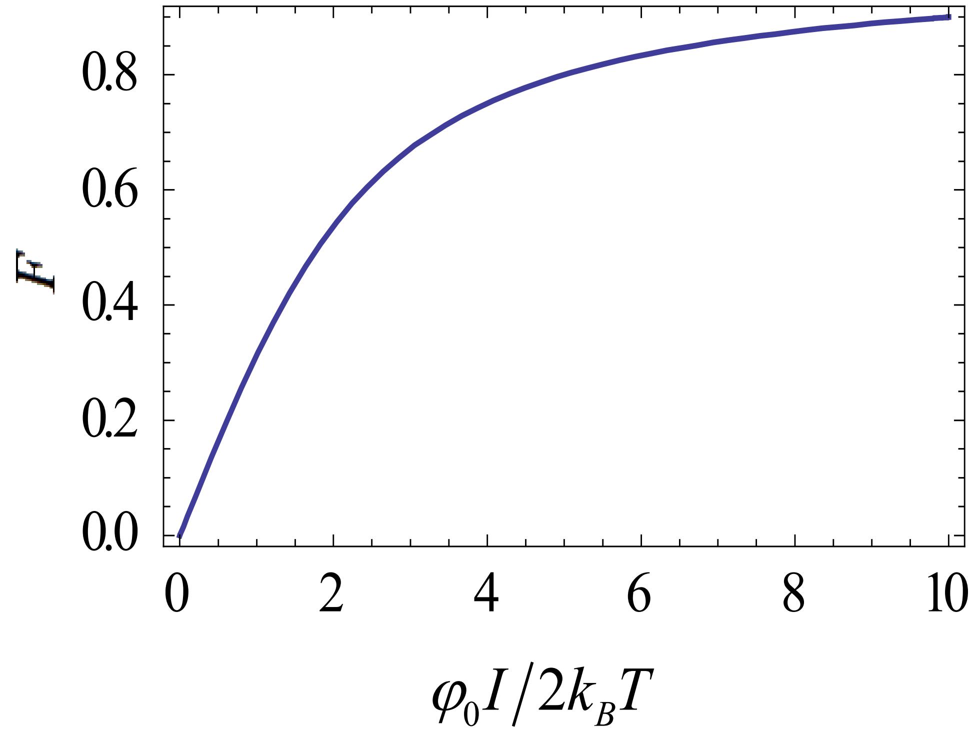
# Thermally activated phase slips

D.S. Golubev and A.D. Zaikin, Phys. Rev. B **78**, 144502 (2008)

$$V = \varphi_0 \Gamma \sinh\left(\frac{\varphi_0 I}{2k_B T}\right)$$

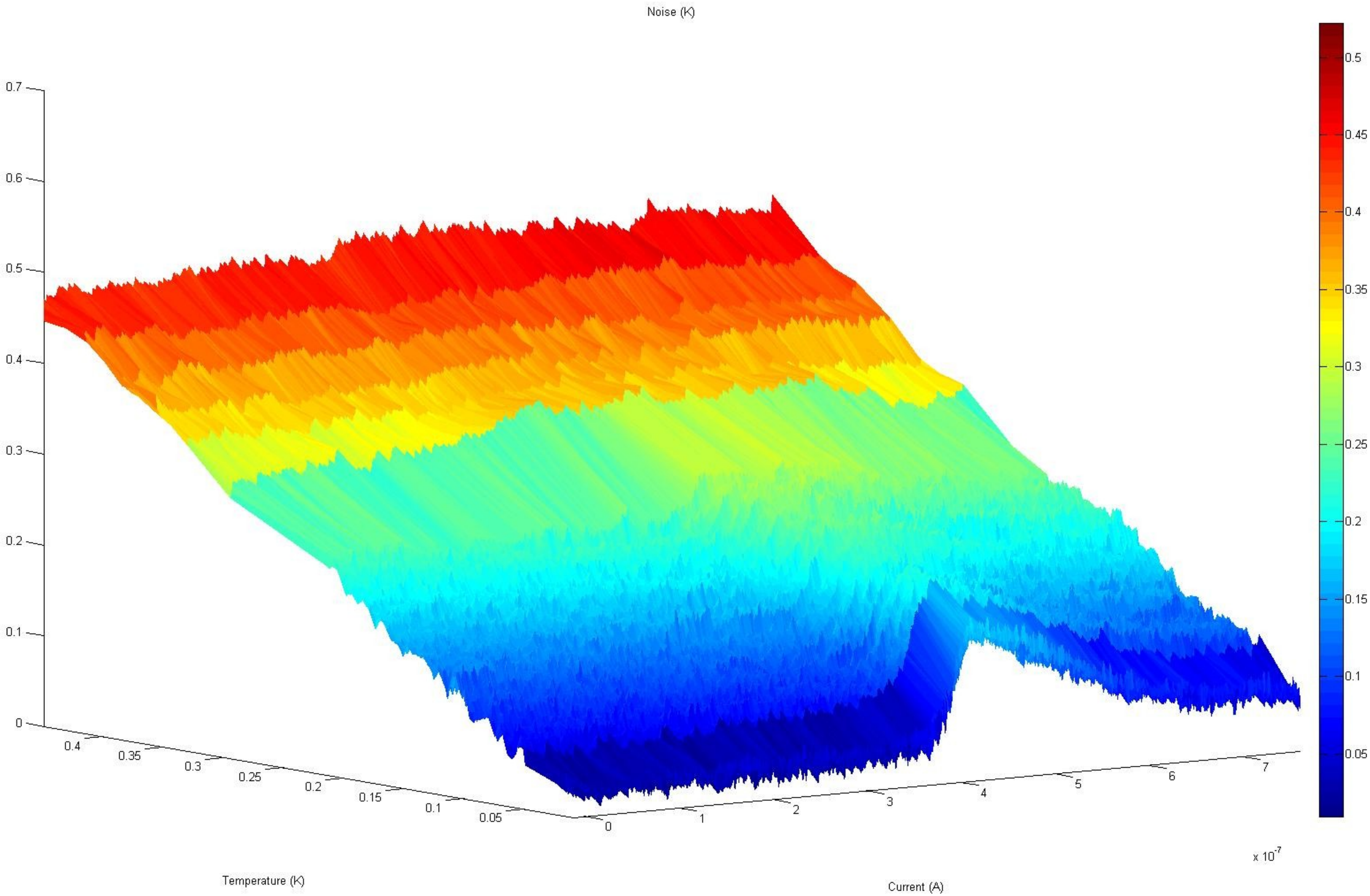
$$S_V = 2\varphi_0^2 \Gamma \cosh\left(\frac{\varphi_0 I}{2k_B T}\right)$$

$$F_V = \frac{S_V}{2\varphi_0 V} = \coth\left(\frac{\varphi_0 I}{2k_B T}\right)$$



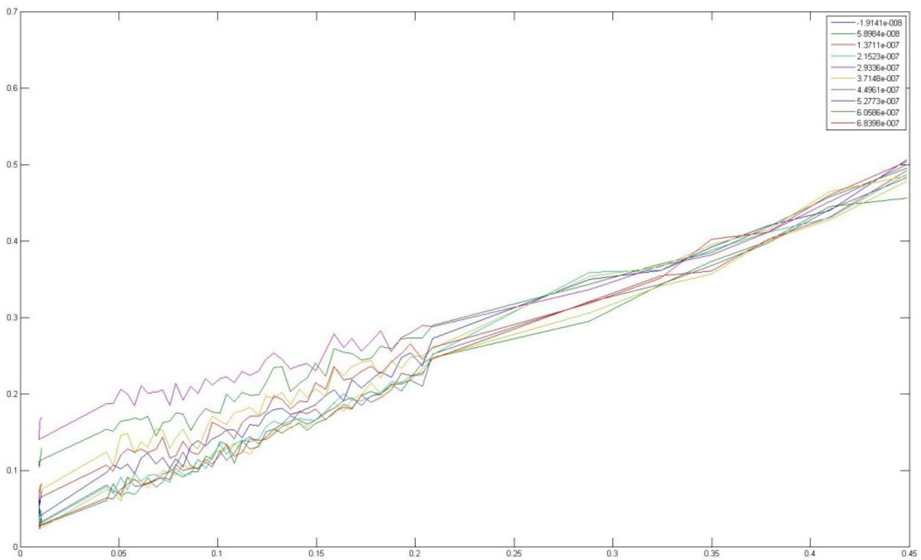

$$F_V = \frac{S_V - S_V(0)}{2\varphi_0 V} = \coth\left(\frac{\varphi_0 I}{2k_B T}\right) - \frac{2k_B T}{\varphi_0 I}$$

# Experimental results: B=0



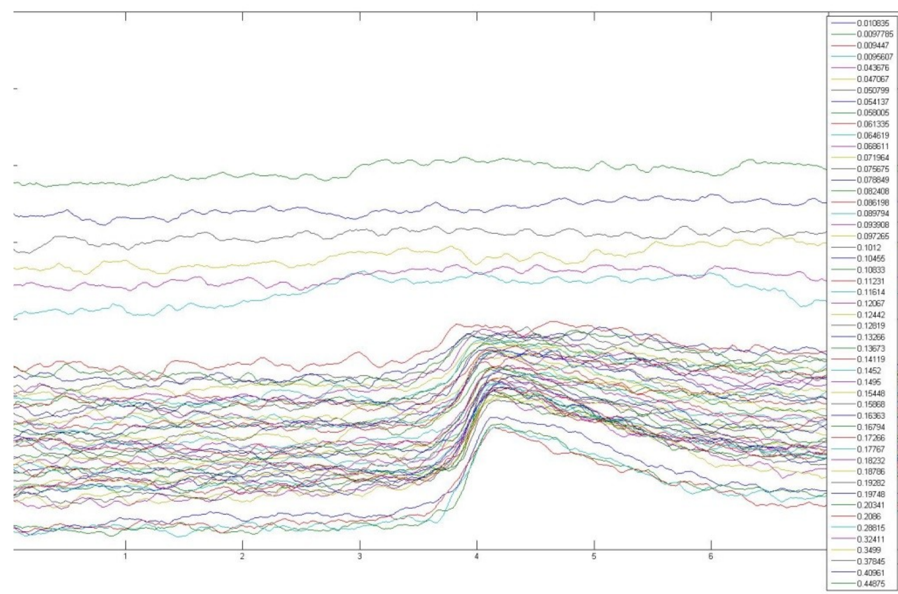
# B=0: 'Slices'

Johnson noise @ various currents



**Finite noise @ T<Tc**

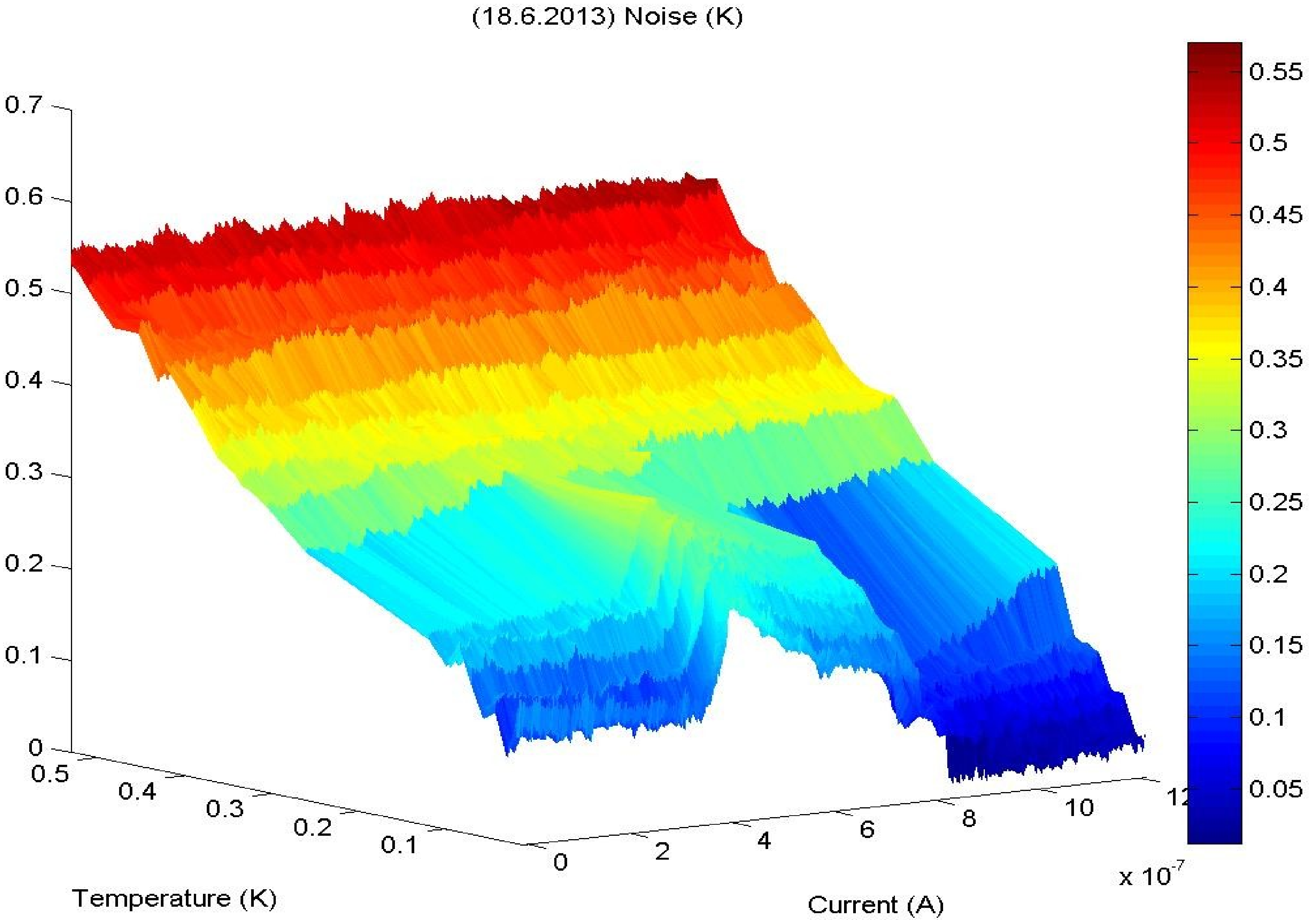
Shot noise @ various temperatures



**Finite noise @ I<Ic**

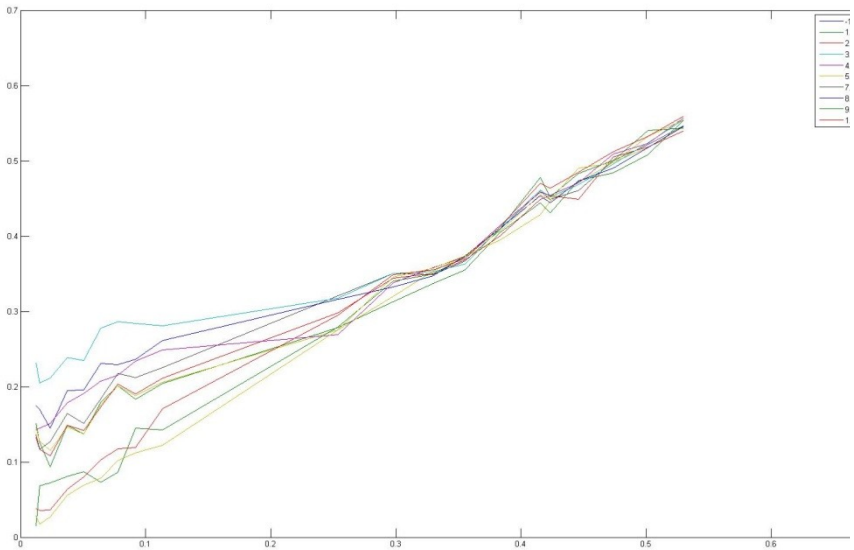
**Jump @ I≈Ic**

# Experimental results: B=35 G

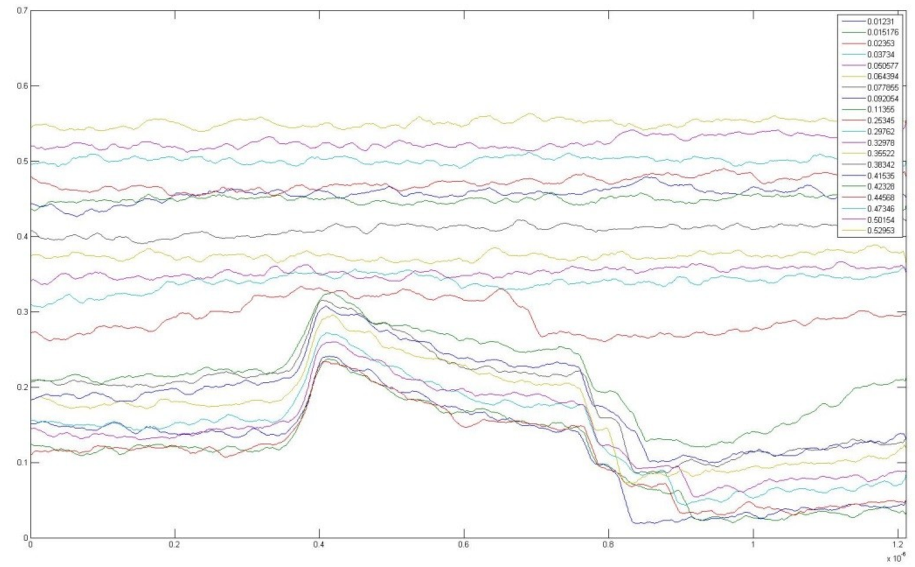


# B=35 G: ‘Slices’

Johnson noise @ various currents



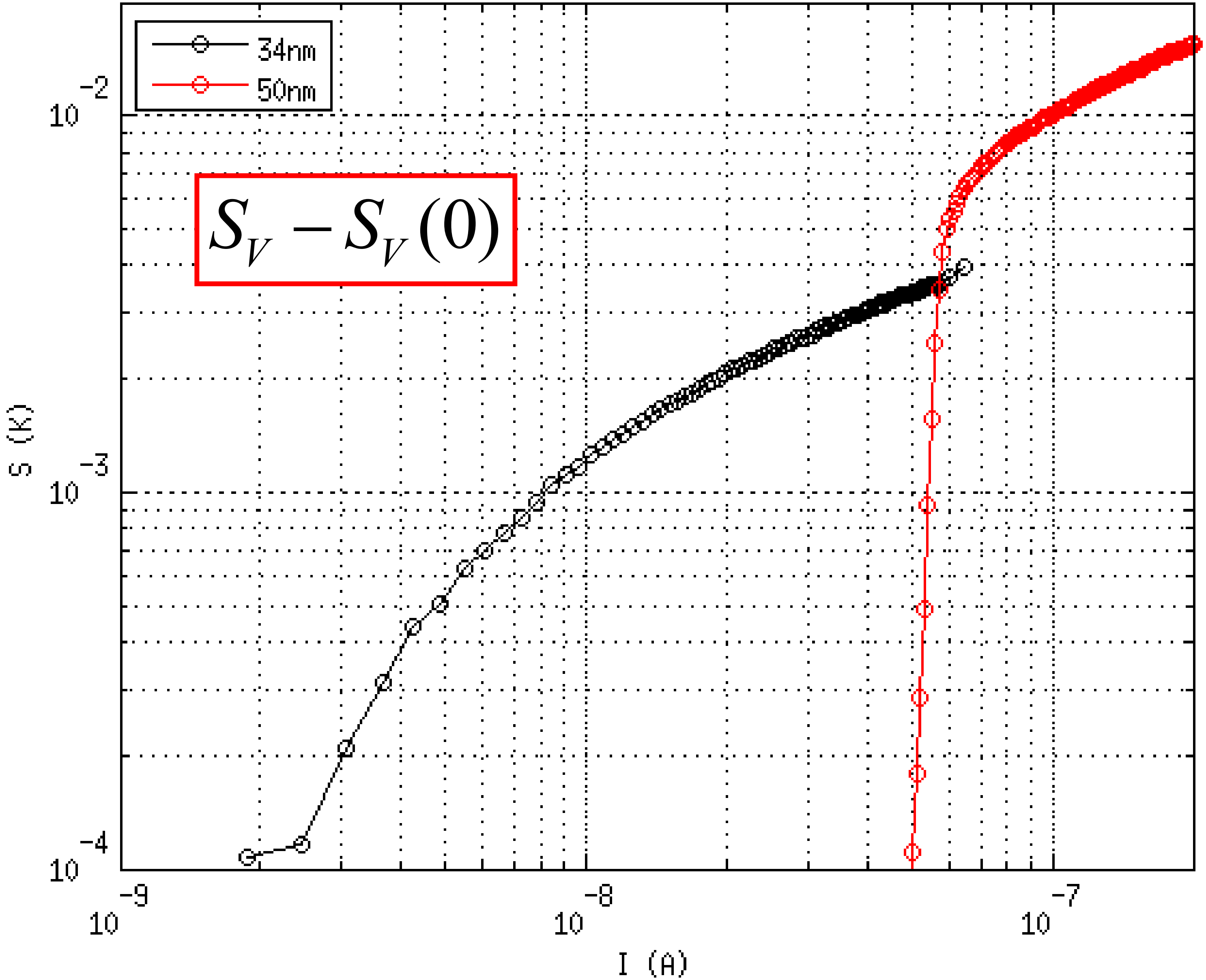

Shot noise @ various temperatures



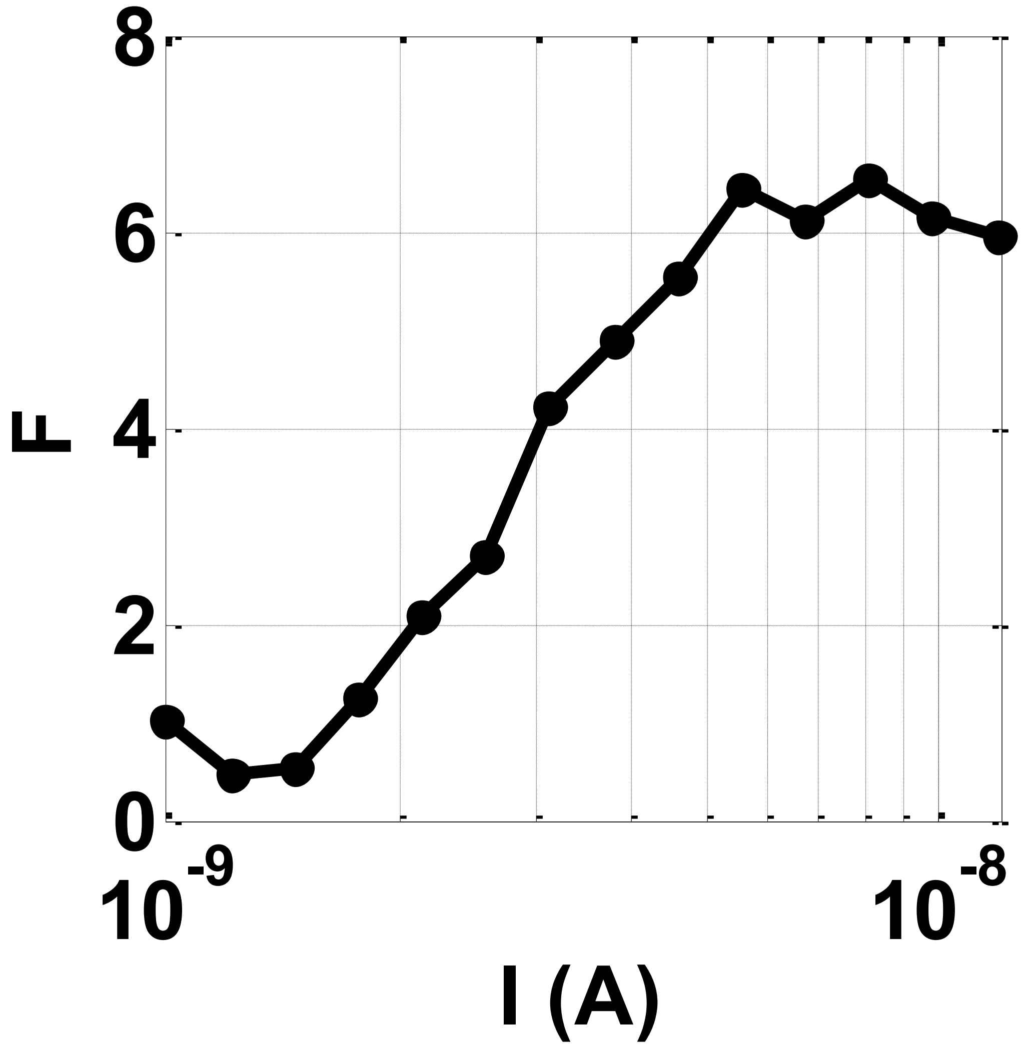

**Finite noise @ T<Tc**
**(not much difference compared to B=0)**

**Finite noise @ I<Ic**
**higher than in normal state!**

# Excess noise vs bias



# Fano factor of 34 nm wire




- Shot noise due to phase slips observed: quantum?
- Shot noise of phase slips close to the Poissonian value
- Above the critical current, large shot noise, Fano ~ 5

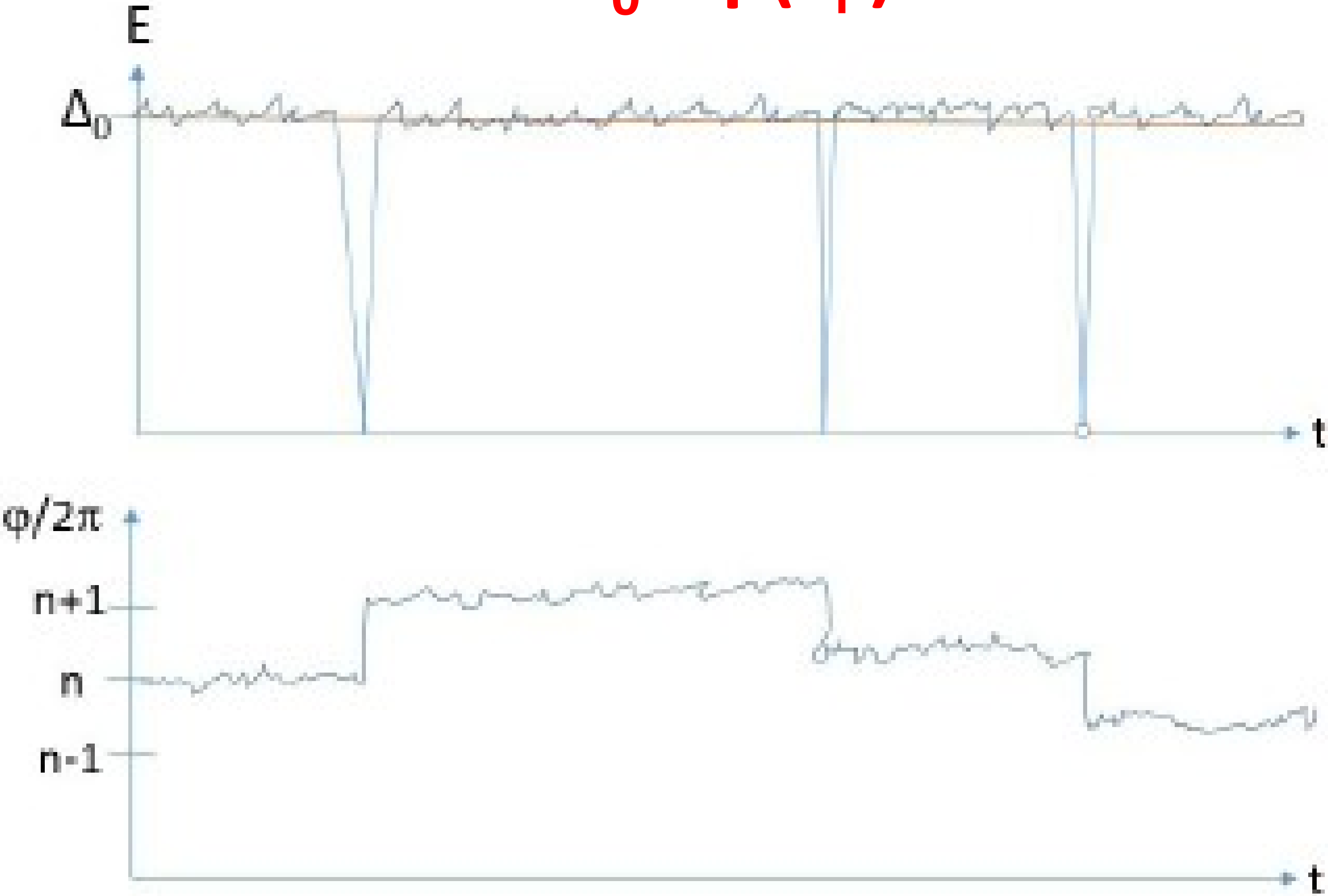
# Conclusion on transport measurements

In aluminium nanowires with diameters below 15 nm in titanium – below 35 nm and in *NbN* below 30 nm quantum fluctuations manifest themselves by broadening the R(T) transition. In the thinnest samples zero resistivity is not reached even at T→0.

In thinnest titanium nanowires finite noise is observed at $T \ll T_c$ and $I \ll I_c$ (?).

# Fluctuations of the superconducting gap amplitude

# Fluctuations of the order parameter $\Delta=\Delta_0 \exp(i\varphi)$

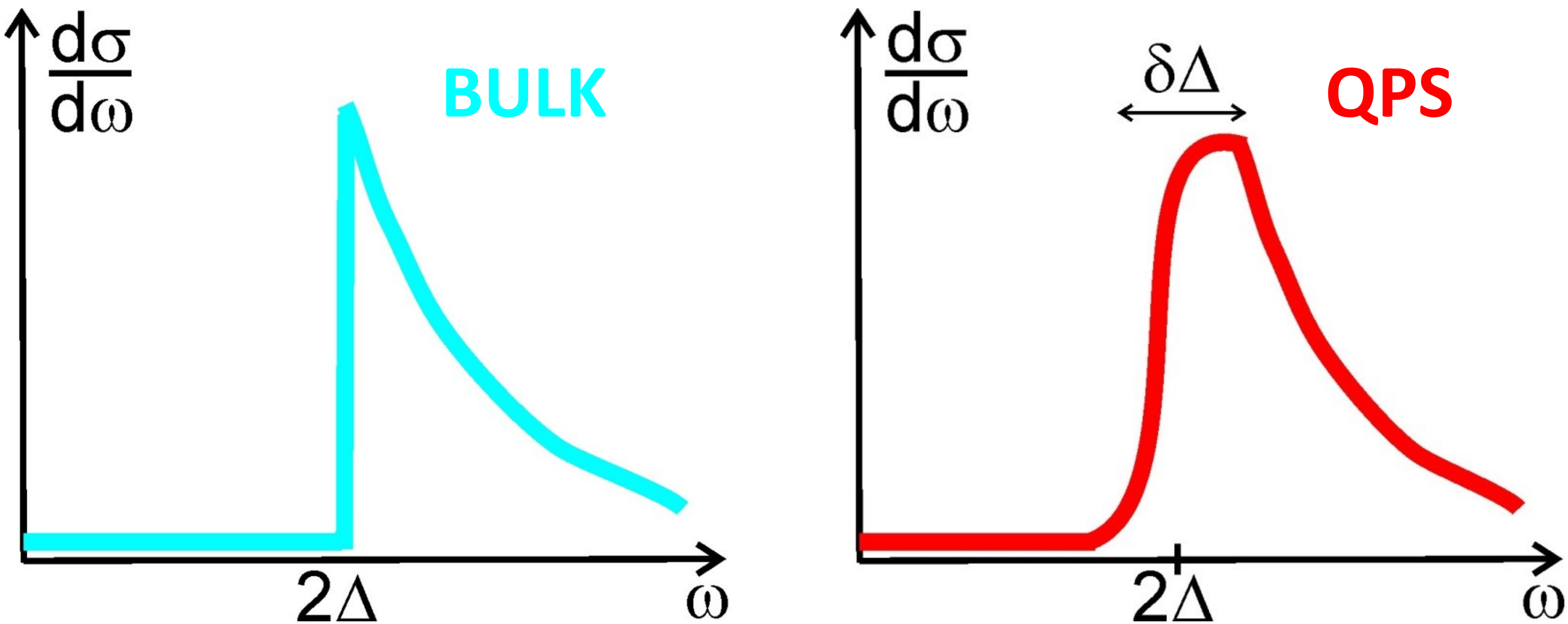



T. Rantala, MSc thesis, University of Jyvaskyla, 2013

# Direct determination of the fluctuating energy gap

$$\delta|\Delta| / |\Delta| \sim (S_{QPS})^{-1}$$

**Superconducting gap can be associated with E/M radiation absorption threshold**



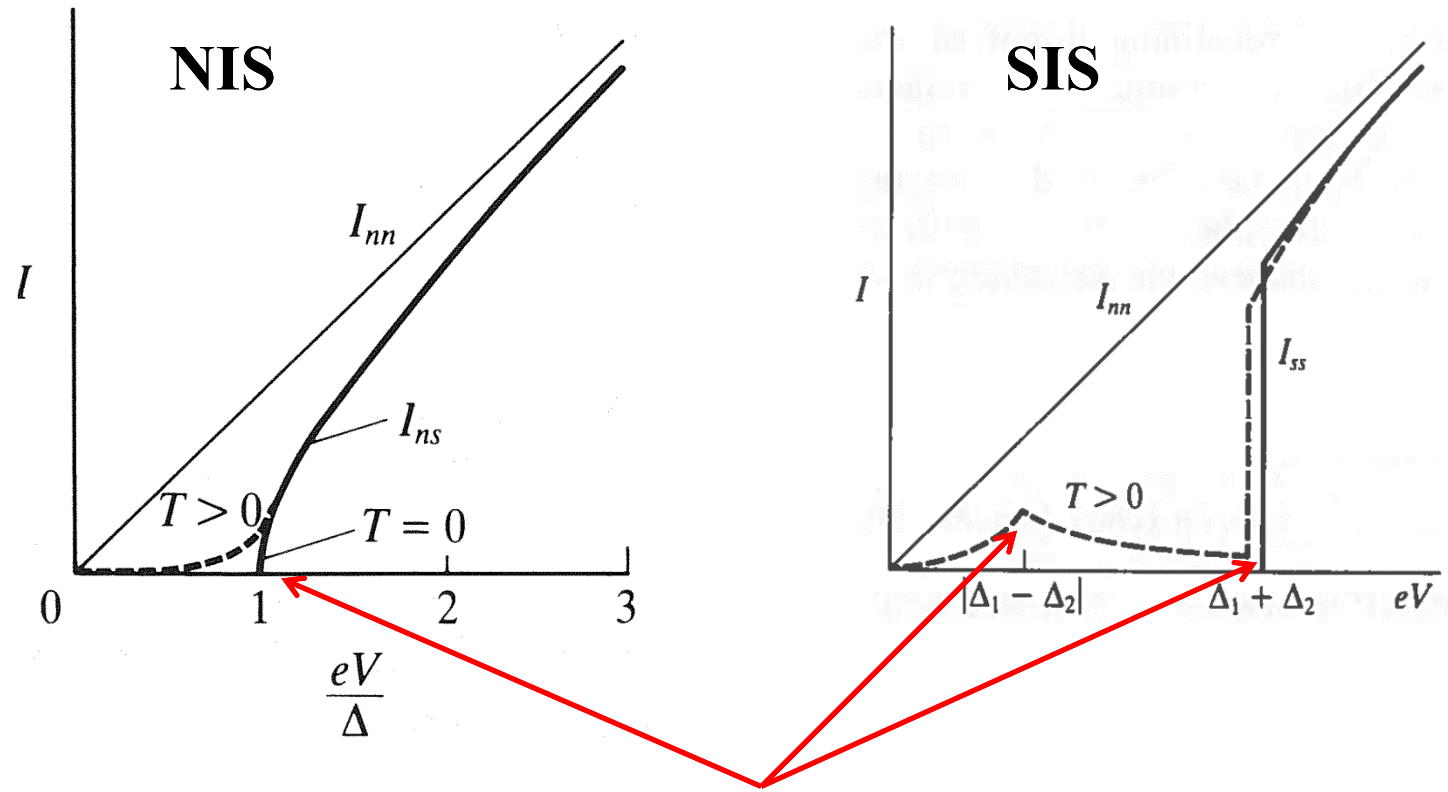

Characteristic scale

Δ(Al)~80 to100 GHz, Δ(Ti)~15 to 25 GHz.

**δΔ /Δ can reach 30% in sufficently thin nanowires**

# Tunneling in NIS and SIS systems

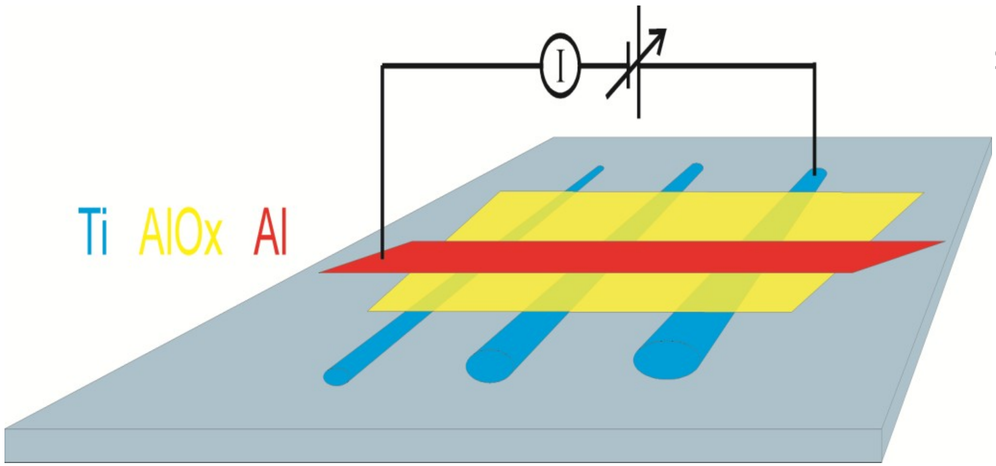
*M. Tinkham, Introduction to supercondcutivity, Mc. Graw-Hill, 1996*

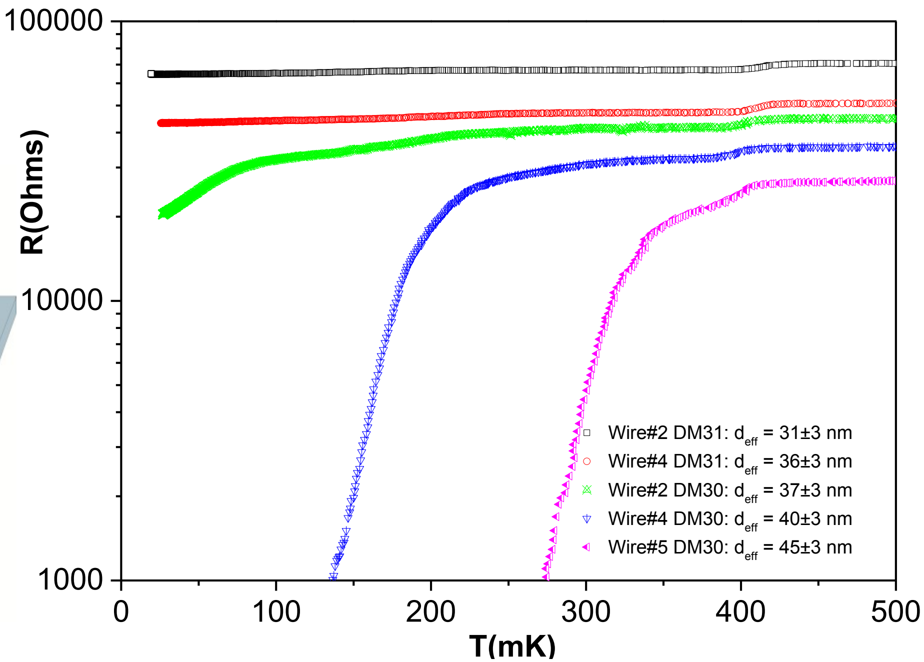



Quantum fluctuations of the gap Δ should lead to extra (size dependent) broadening of the gap edge

# Experiment

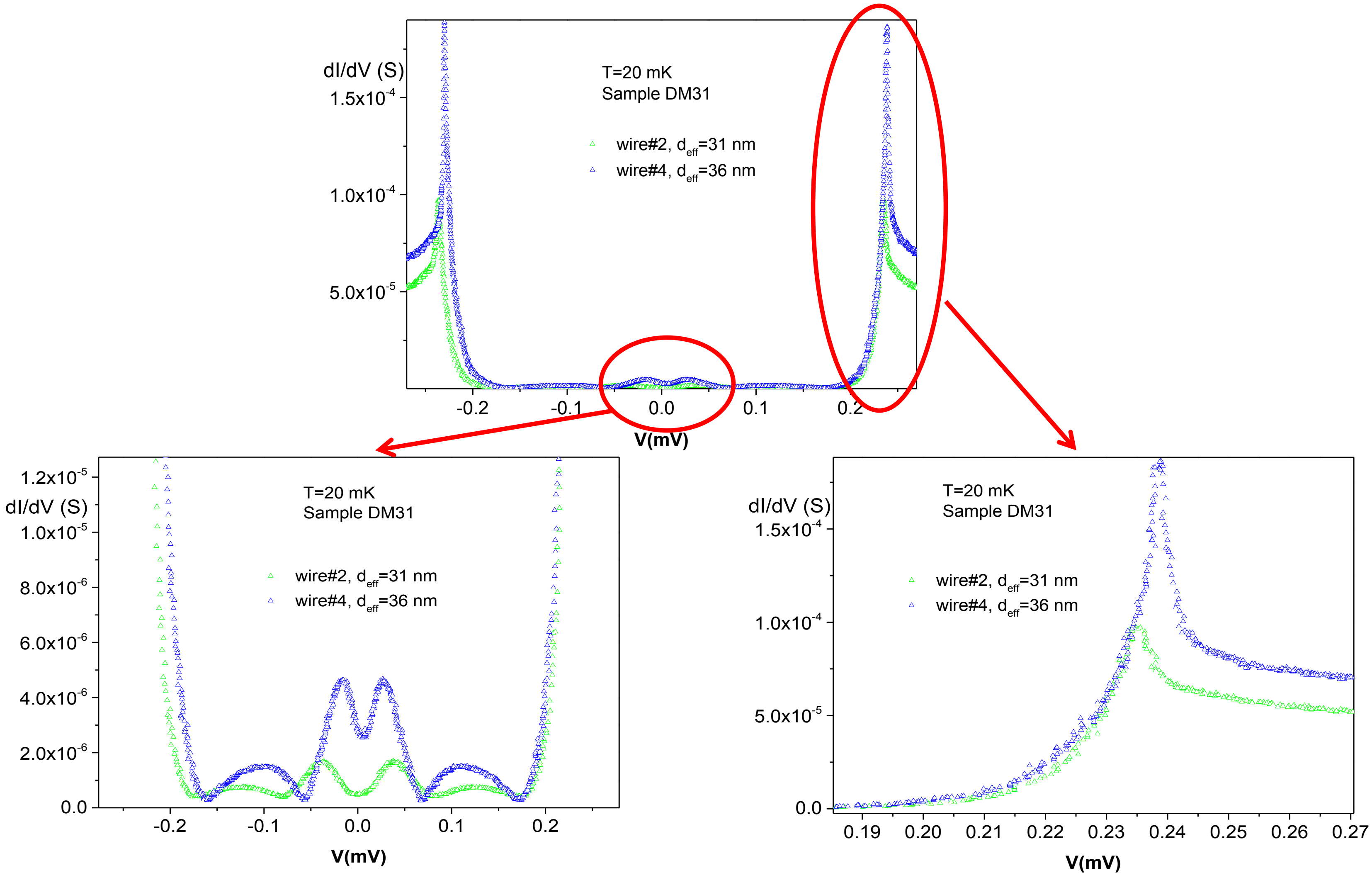
Schematic of the experiment




**R(T) of each nanowire and I(V) chartacteristic between the nanowire and the aluminium electrode can be measured independently.**
**R(T) dependencies at these diameters are broad associated with QPS.**

# SIS I(V) at various diameters of nanowires



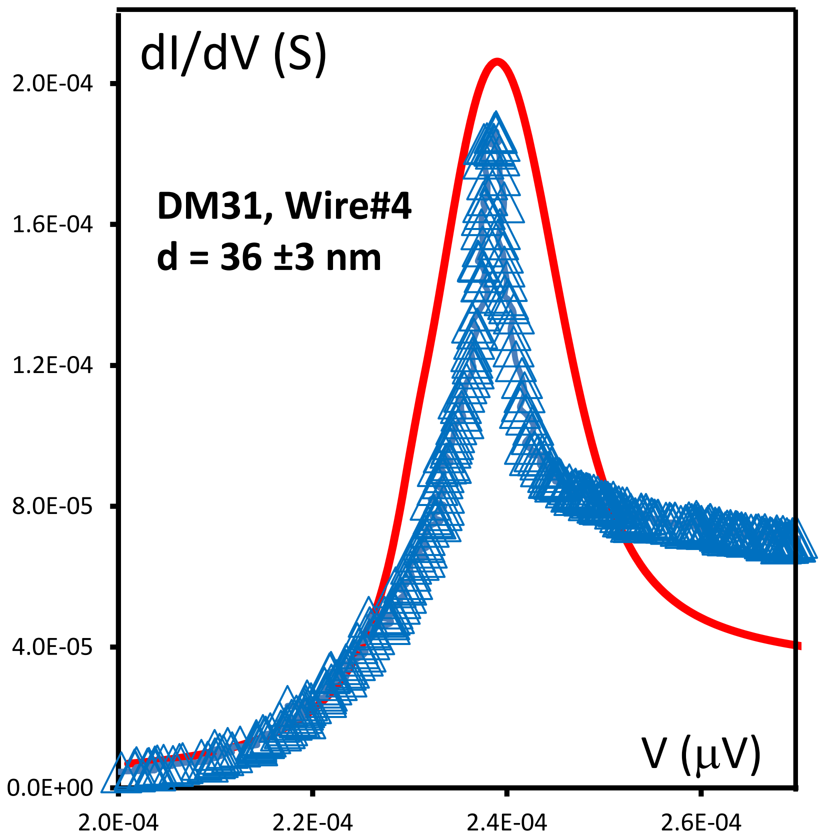

$\Delta 1 - \Delta 2$ features and
Coulomb blockade and/or Josephson current (?)

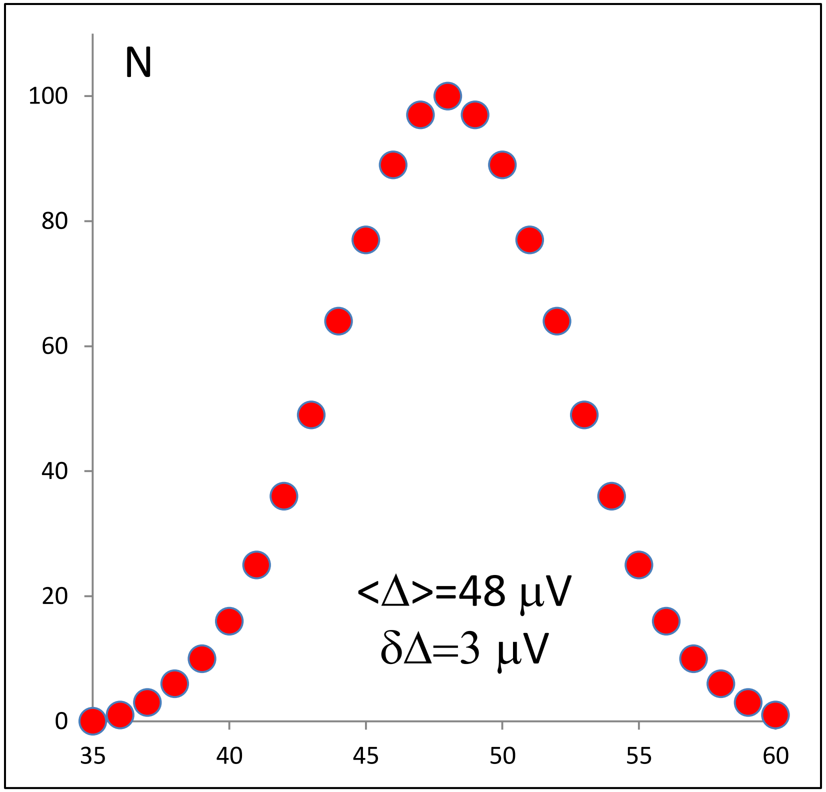
$\Delta 1 + \Delta 2$ features

**The smaller the nanowire diameter (1) the smaller the average value $\langle \Delta \rangle$ and (2) the larger the gap smearing $\delta\Delta$.**

# Simulations

We simulate I(V) dependencies assuming Gaussian distribution of the gap fluctuations centered at <Δ> with standard deviation δΔ.




**Limitations & postulates:**

- **Gaussian distribution of quantum fluctuation amplitude**
- **Cut-off $\Delta_{bulk}/2 < \Delta < \Delta_{bulk}$**
- **Finite magnitude of voltage modulation and finite lock-in integration time**

# Conclusions on physics

Quantum fluctuations in narrow superconducting nanowires suppress ‘basic’ superconducting attributes manifesting as:

- Electron transport:

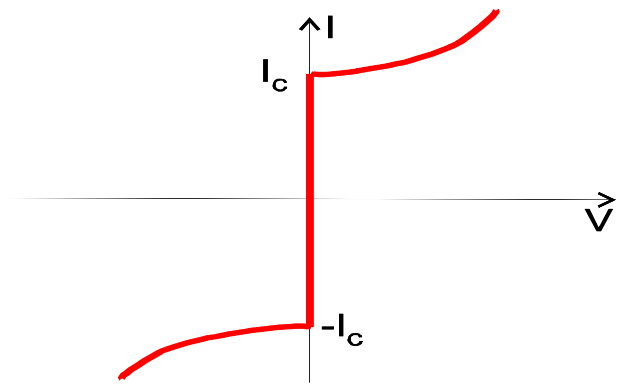
  (1) Broadening of the R(T) transition. In thinnest samples zero resistance is not reached even at $T \rightarrow 0$.

  (2) Finite noise at $T \ll T_c$ and $I \ll I_c$.

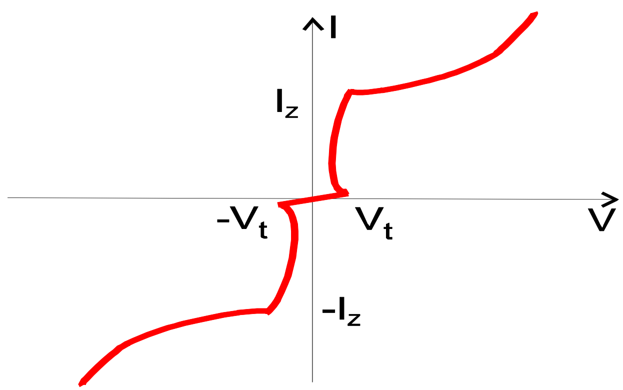
- Suppression of persistent currents in nanorings.
- Smearing of the superconducting gap edge.

# Applications

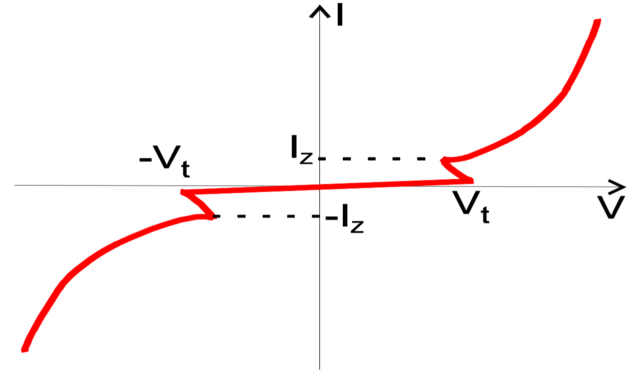
# Coulomb Effects




Thick (conventional) superconducting wire

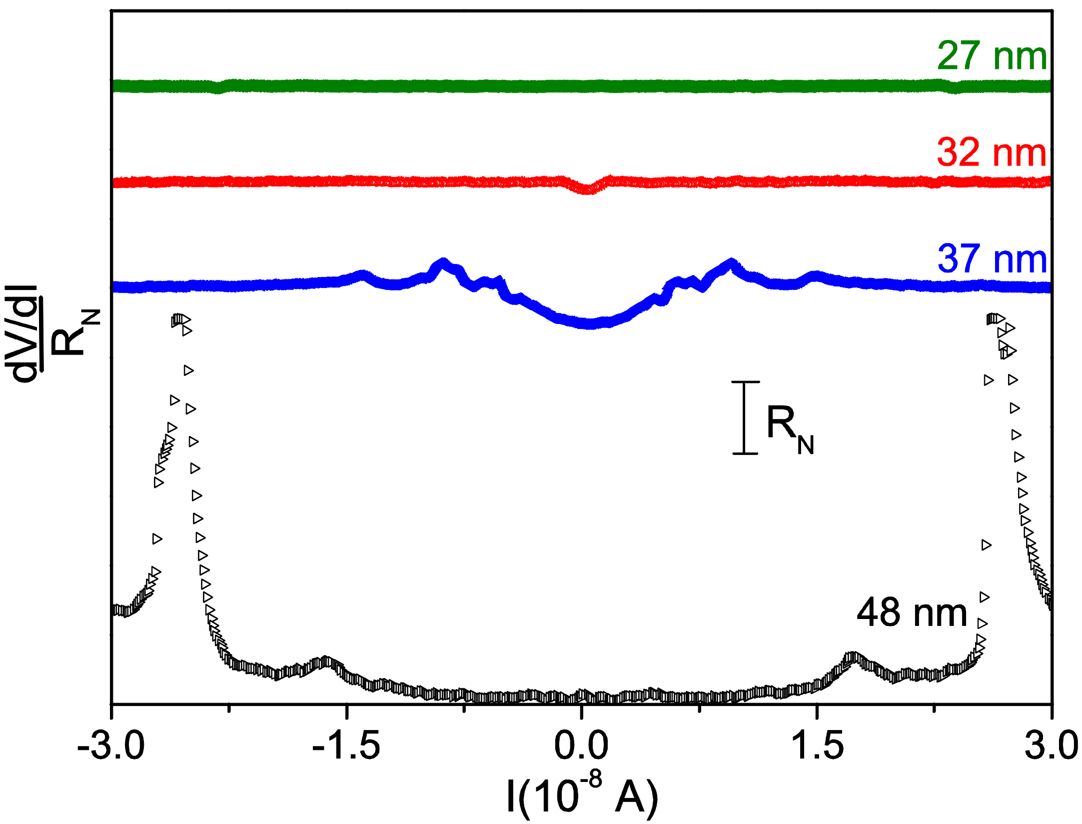



Thin wire and low-Ω environment




Very thin wire and high-Ω environment: Coulomb blockade

# Ti nanowire in low-Ohmic environment




*Differential resistance dV/dI, normalized by the normal state resistance $R_N$, as function of the bias current I.*

In 'thick' samples one observes the zero resistance state below the critical current. With reduction of the wire diameter the 'residual' critical current disappears and the finite resistance is observed .

# Quantum duality between JJ and QPS junctions

**Hamiltonian of a supercondcuting nanowire in the regime of quantum fluctuations:**

$$\widehat{H} = \frac{E_L}{(2\pi)^2}\widehat{\phi}^2 - E_{QPS}\cos(2\pi\widehat{q}) + \widehat{H}_{coup} + \widehat{H}_{env}$$

**is dual to the corresponding Hamiltonian of a Josephson junction:**

$$\widehat{H} = E_C\widehat{q}^2 - E_J\cos(\widehat{\varphi}) + \widehat{H}_{coup} + \widehat{H}_{env}$$

**with the accuracy of substitution:** $E_C \leftrightarrow E_L, E_J \leftrightarrow E_{QPS}, \varphi \leftrightarrow \pi q / 2e$

**$E_L$ , $E_C$ , $E_J$ and $E_{QPS}$ are the inductive, charging, Josephson coupling and QPS energies, φ is phase and q is quasicharge.**

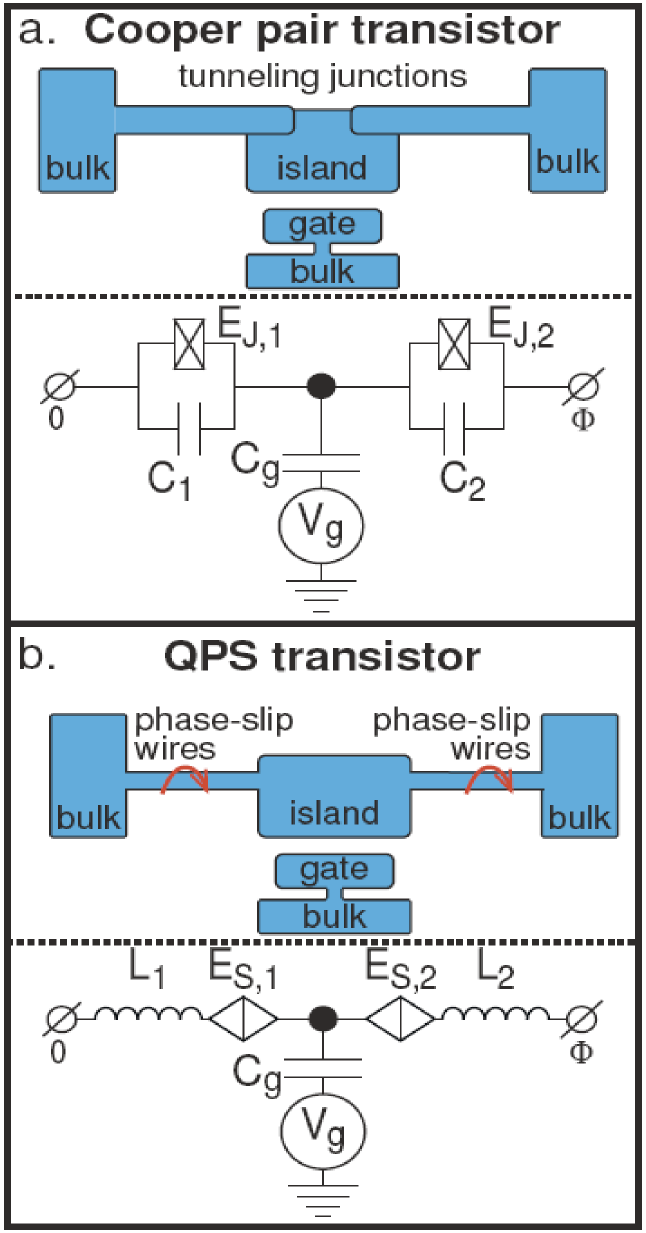
**The extensively developed physics for Josephson systems can be 'mapped' on the superconducting nanowires in the regime of quantum fluctuations.**


*D. V. Averin and A. A. Odintsov, Phys. Lett. A* ***140*** *(1989) 251*

*J. E. Mooij and Yu. V. Nazarov, Nature Physics* ***2*** *(2006) 169*

A. M. Hriscu and Yu. V. Nazarov. *Phys. Rev. B* 83, 174511 (2011)

# Cooper pair transistor without a tunnel junction




*A.M. Hriscu and Yu. V. Nazarov. Phys. Rev. B 83, 174511 (2011)*

Let us consider the simplest case of a QPS Copper pair box:

$$\hat{H}_{QPSbox} = E_c\left(\hat{Q} - \frac{q}{2e}\right)^2 + E_L\hat{\phi}^2 - E_{QPS}\cos(2\pi\hat{Q})$$

First two terms correspond to linear LC oscillator. If to shift the charge variable by q/2e, the induced charge disappears from the oscillator part, while the QPS amplitudes acquire the phase factors:

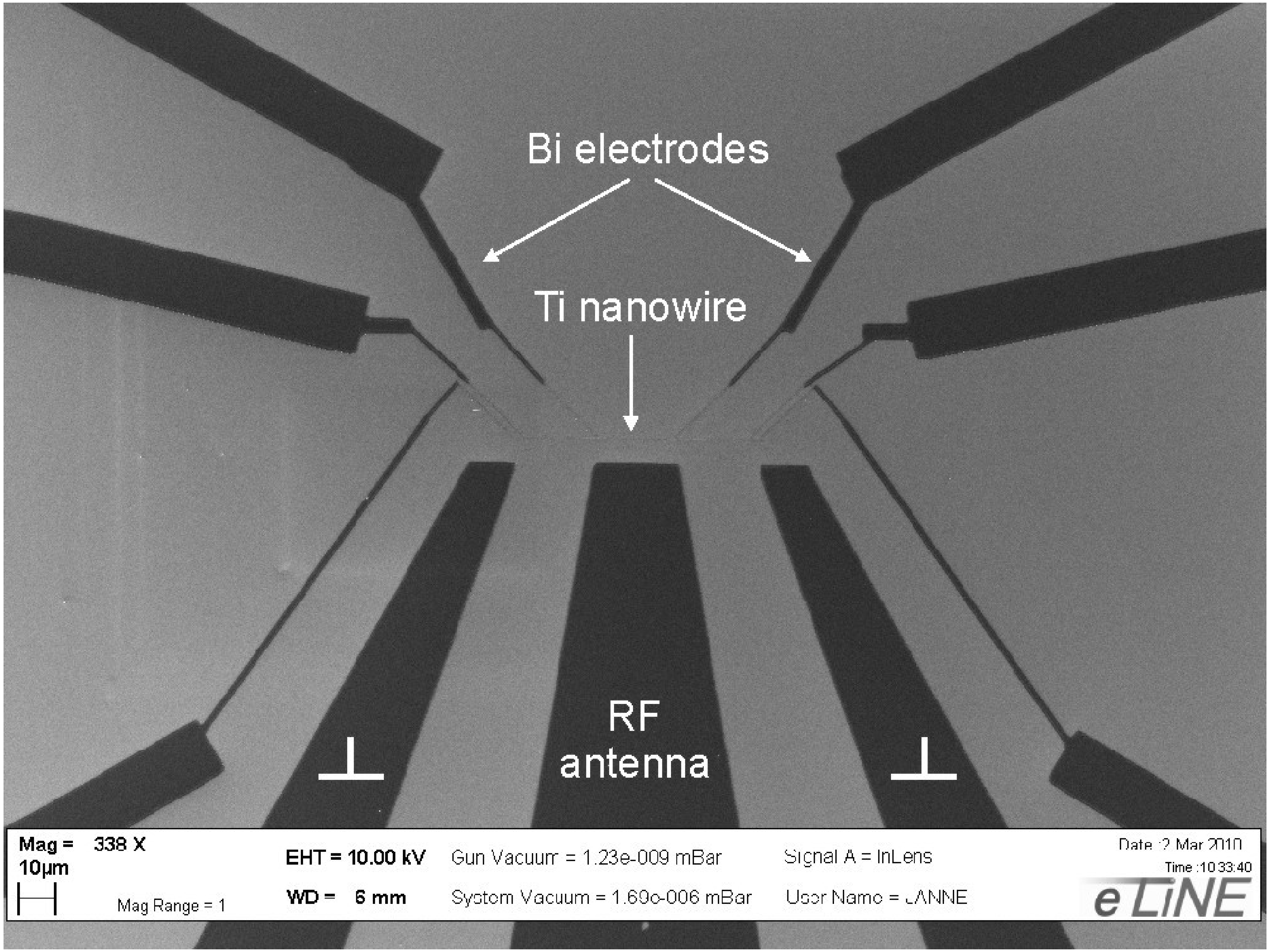
$$\hat{H}_{QPS}\psi(\phi) = -E_{QPS}e^{-\frac{i\pi q}{e}}\psi(\phi + 2\pi) - E_{QPS}e^{+\frac{i\pi q}{e}}\psi(\phi - 2\pi)$$

**The charge sensitivity is purely due to the coherent QPS term: the induced charge q/2e affects the interference of the phase slips with two opposite directions ±2π.**

The mandatory requirement for the charge effects observation is the high impedance of the environment: current bias→ charge is a 'good' quantum number.

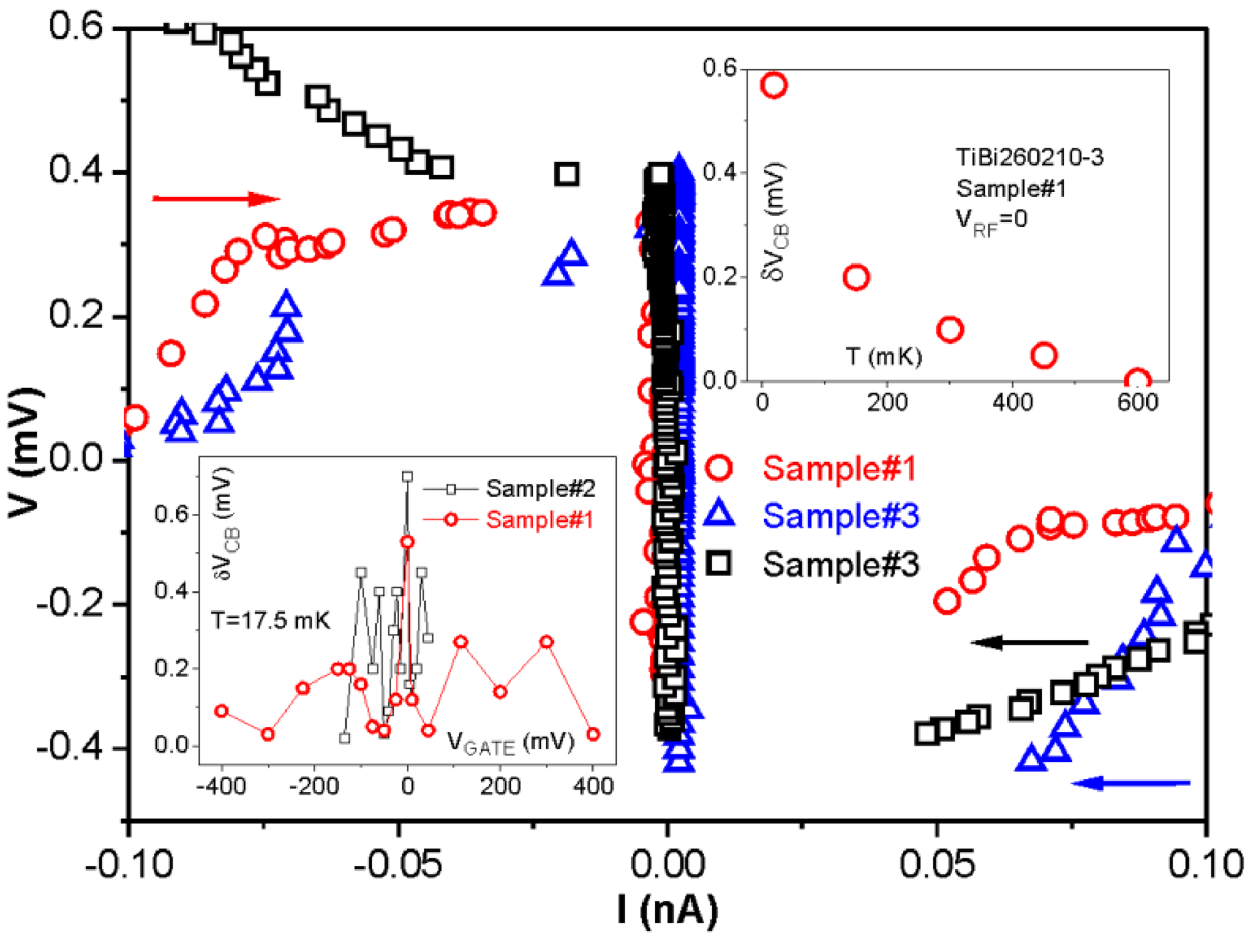
**QPS is a dynamic equivalent of a conventional (static) tunnel junction.**

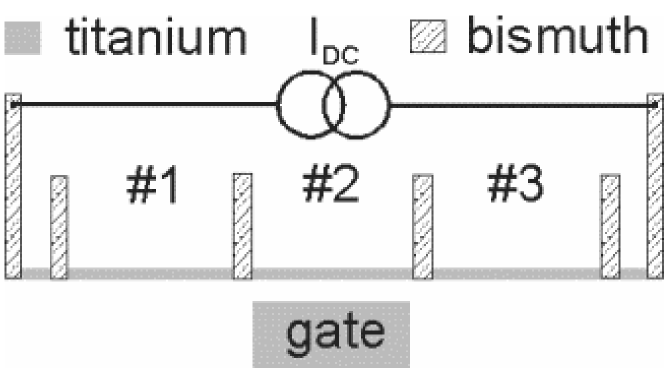
# High-Ohmic environment




Purely dissipative environment:
high-Ohmic normal metal probes with $R_{probe}$ up to 10 MOhm

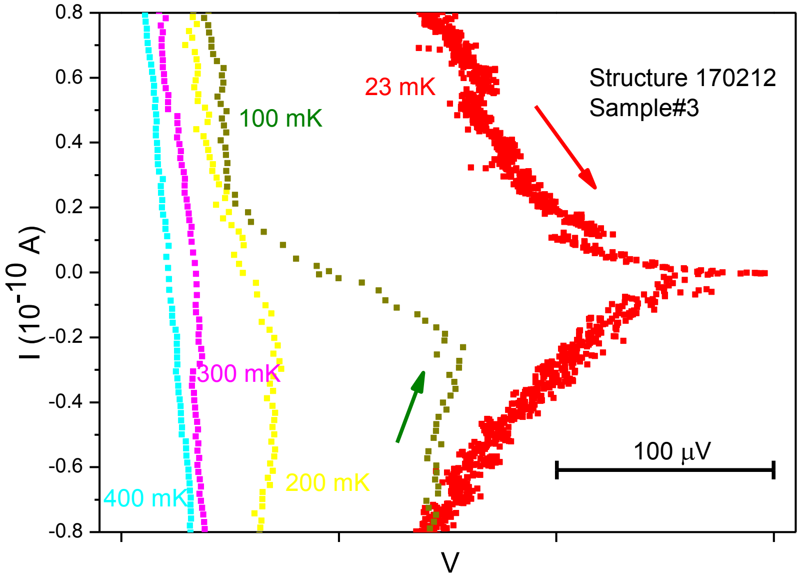
# 24 nm titanium nanowire, 10 MΩ contacts

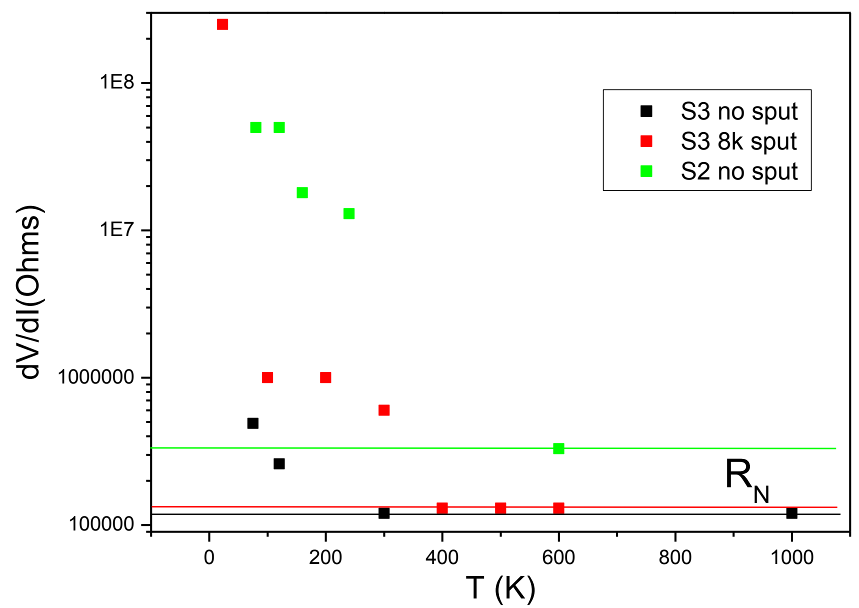


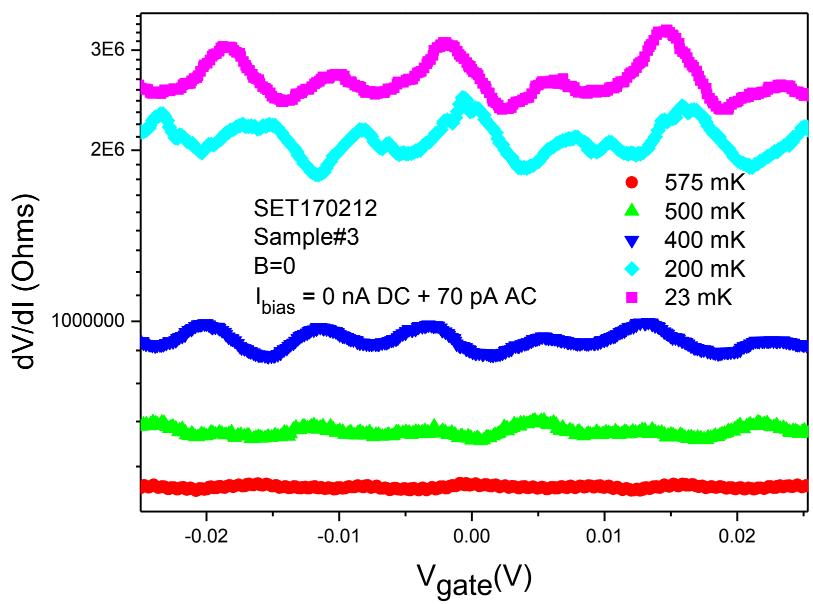




All three neighboring parts of the same multiterminal structure demonstrate the same value of the Coulomb gap. The effect disappears above $T_c$ and/or $H_c$.

# Temperature dependencies

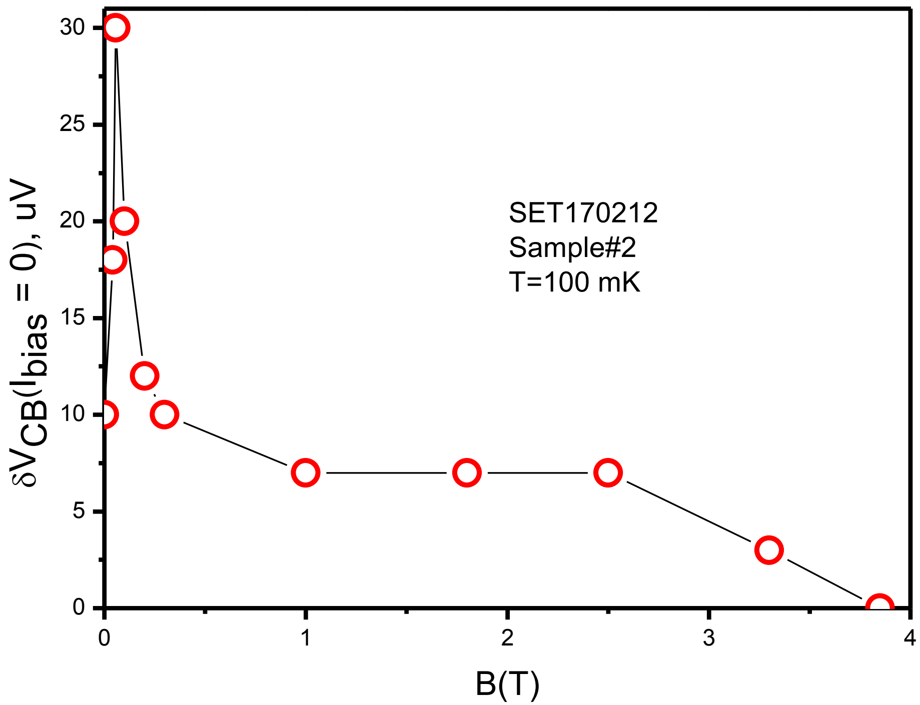


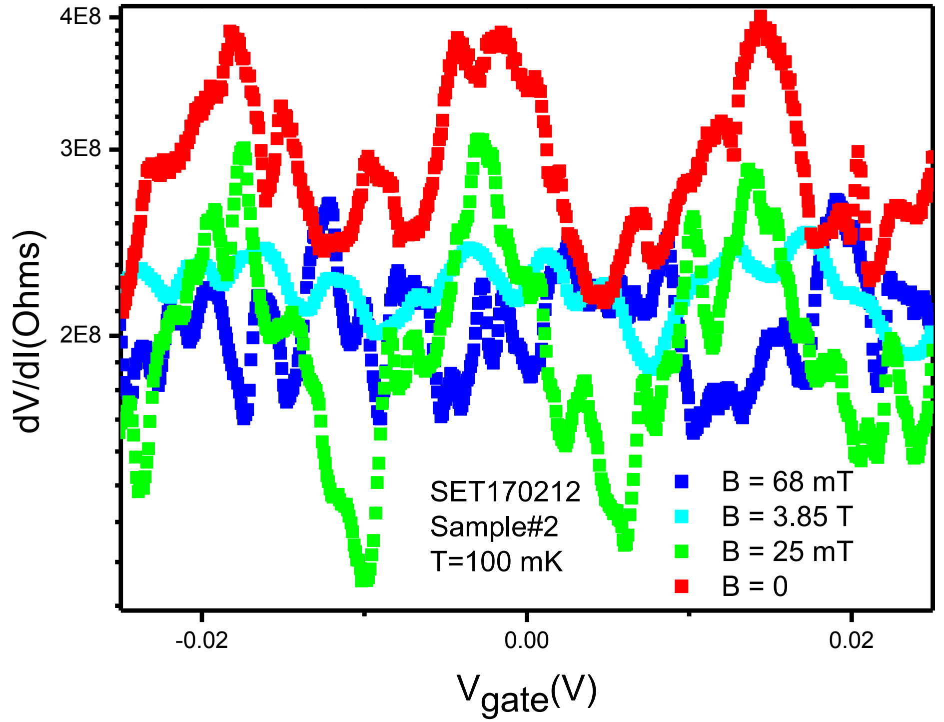




The Coulomb gap and the gate modulation disappear above 450 mK (the $T_c$ of Ti?).

# Magnetic field dependencies



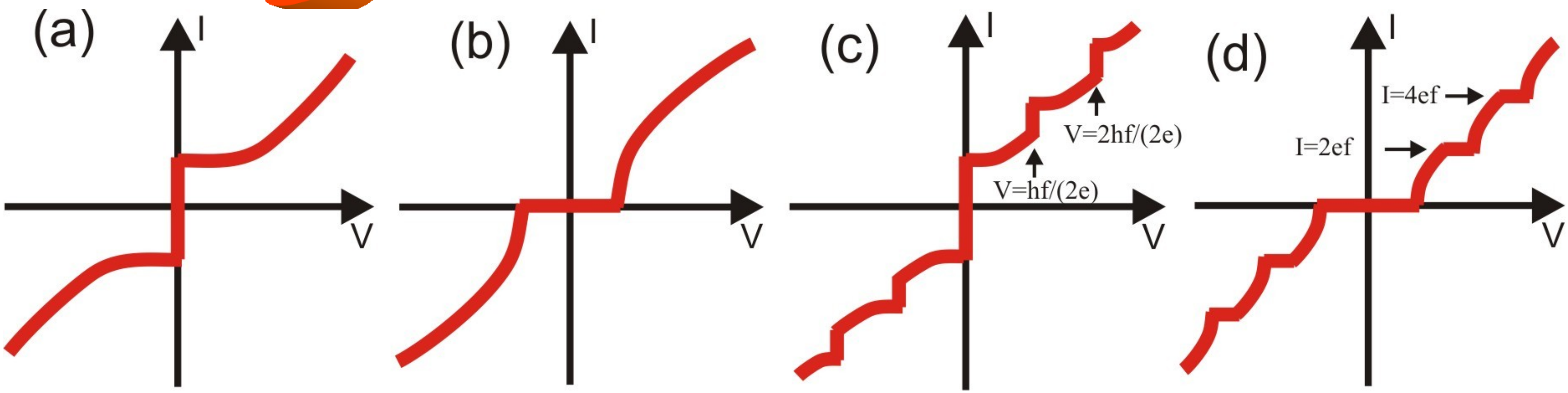

At a given (low) temperature the Coulomb gap and the gate modulation disappear above certain magnetic field (the $B_c$ of Ti?).

# Digest on QPS transistor

- Quantum phase slip and Josephson tunneling are the phenomena described by identical Hamiltonians: the quantum dynamics is indistinguishable.
- Supercondcuting nanowire (homogeneous!), in the regime of QPS, is the dynamic equivalent of a Josephson junction.
- Containing no static (in space and time) junctions, QPS nanowire can sustain much higher currents and has no undesired two-level ’fluctuators’ present in tunnel contacts.
- All phenomena, observable in Josephson systems, can be observed in QPS nanowires.

**Electron transport in a Cooper pair transistor is a <u>periodic</u> process corresponding to cyclic charging/discharging of the central island by *2e*. Syncronisation of the process with external drive should lead to resonance.**

# Bloch oscillations



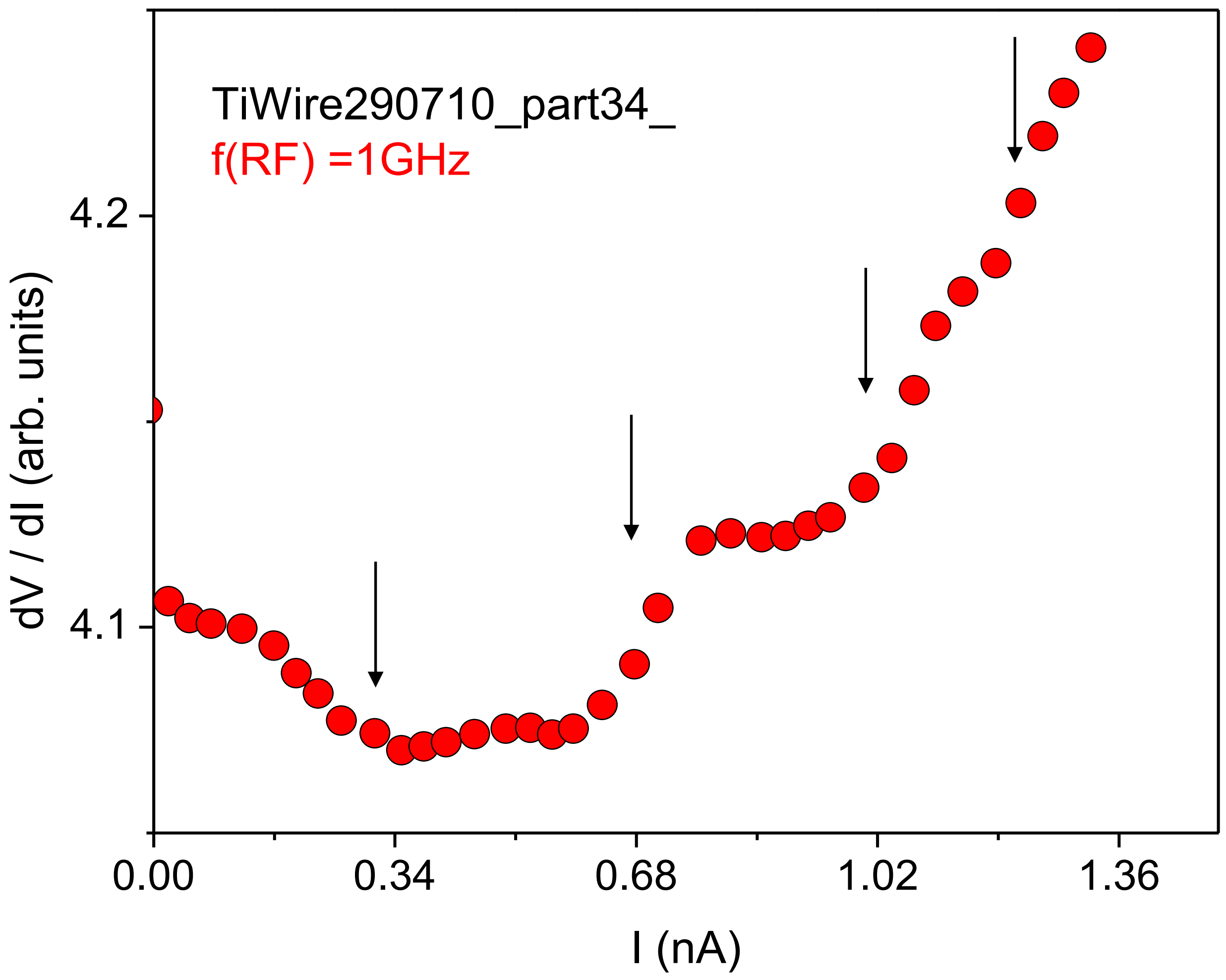

Josephson junction: critical current

QPS wire: critical voltage

Josephson junction: voltage steps (Shapiro effect)

QPS wire: current steps

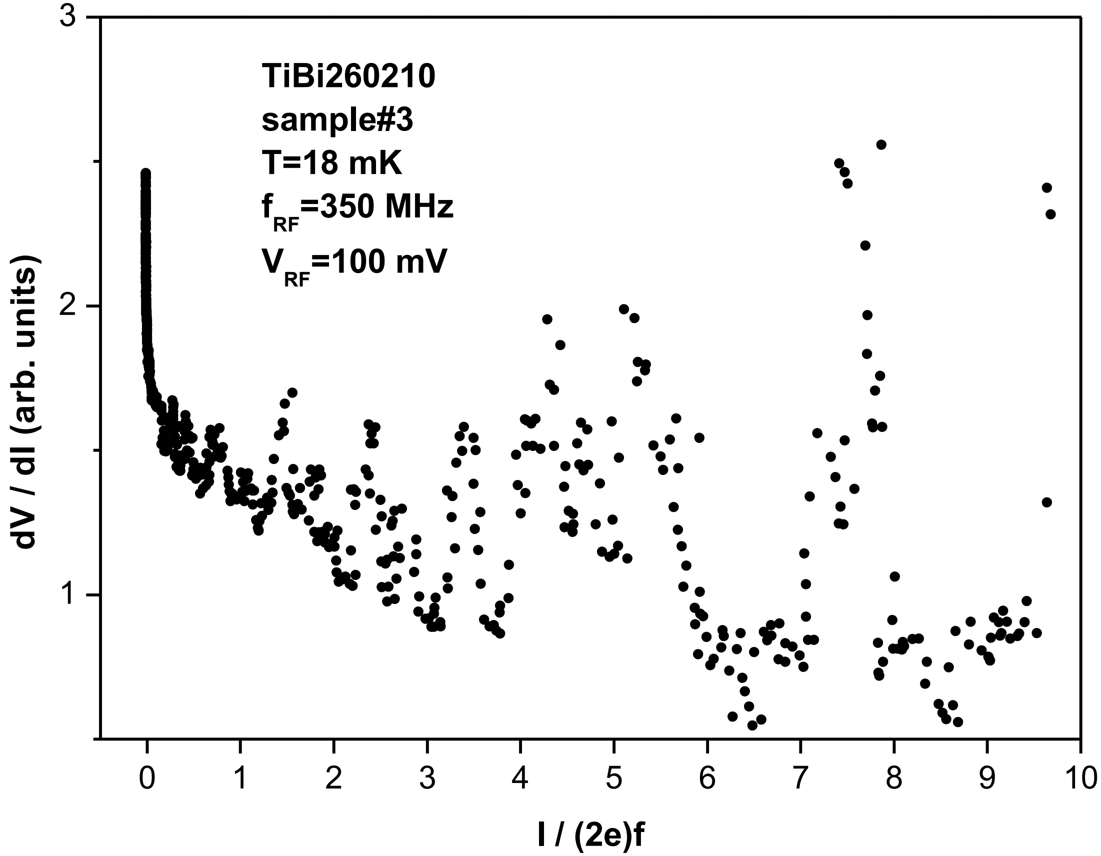
# Not-too-high-Ω environment (50 kΩ)




Arrows mark the prediction

$I = n \times 2e \times f_{RF}$

# Ti-nanowire with high-Ohmic environment

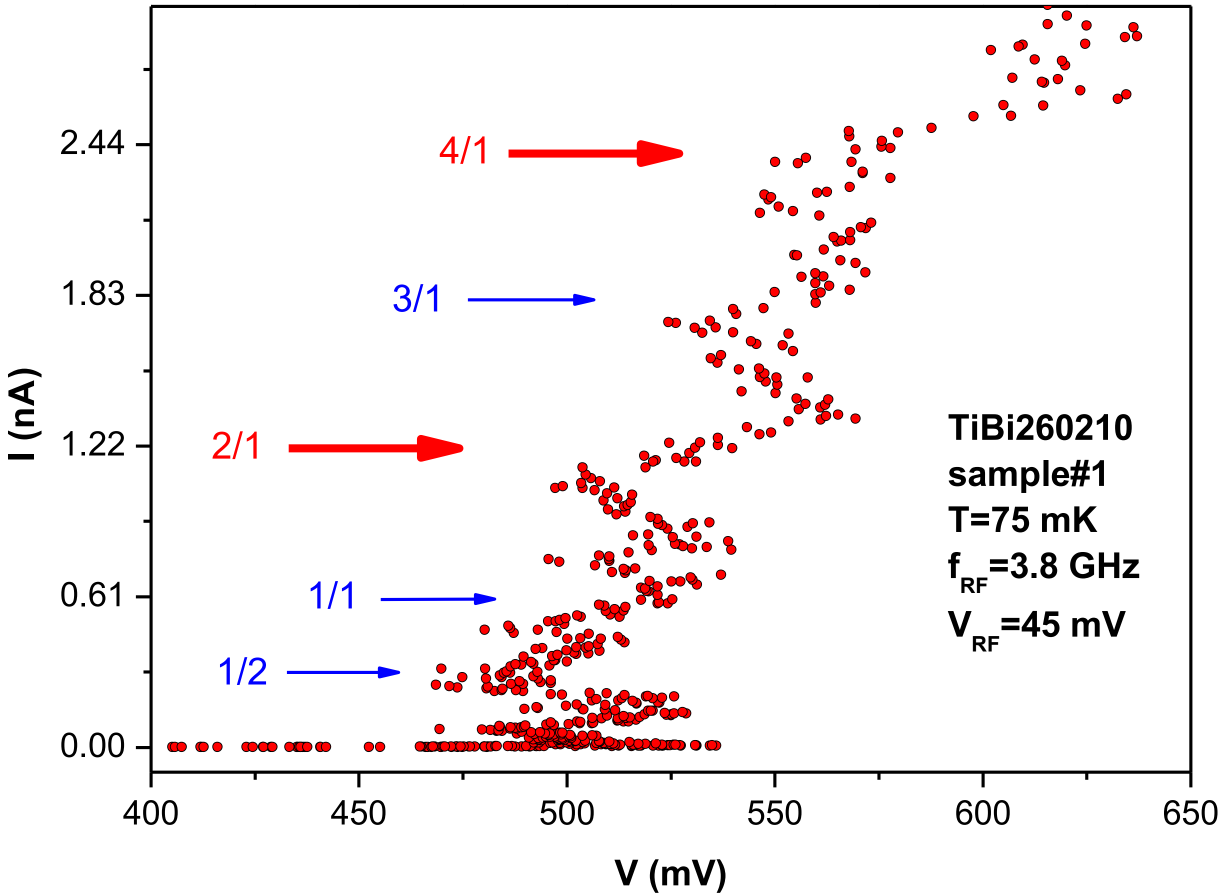



*dV/dI at $f_{RF}$=350 MHz and amplitude 100 mV. Current is normalized by (2e)×$f_{RF}$. One can distingush steps with quantum number n≤8.*

**Universal relation $I(n) = (2e)*n*f_{RF}$**

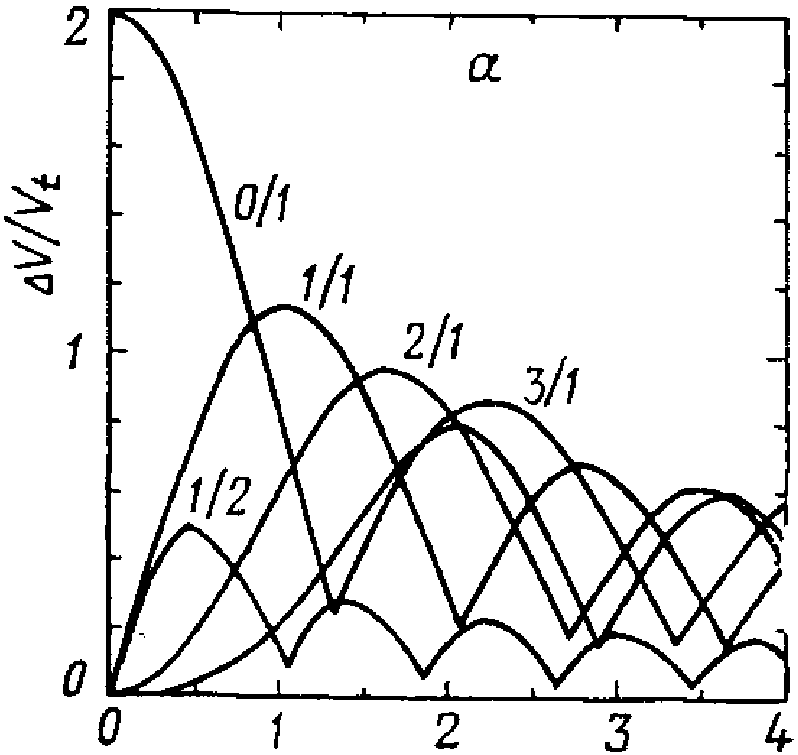
# RF-induced steps at high frequency

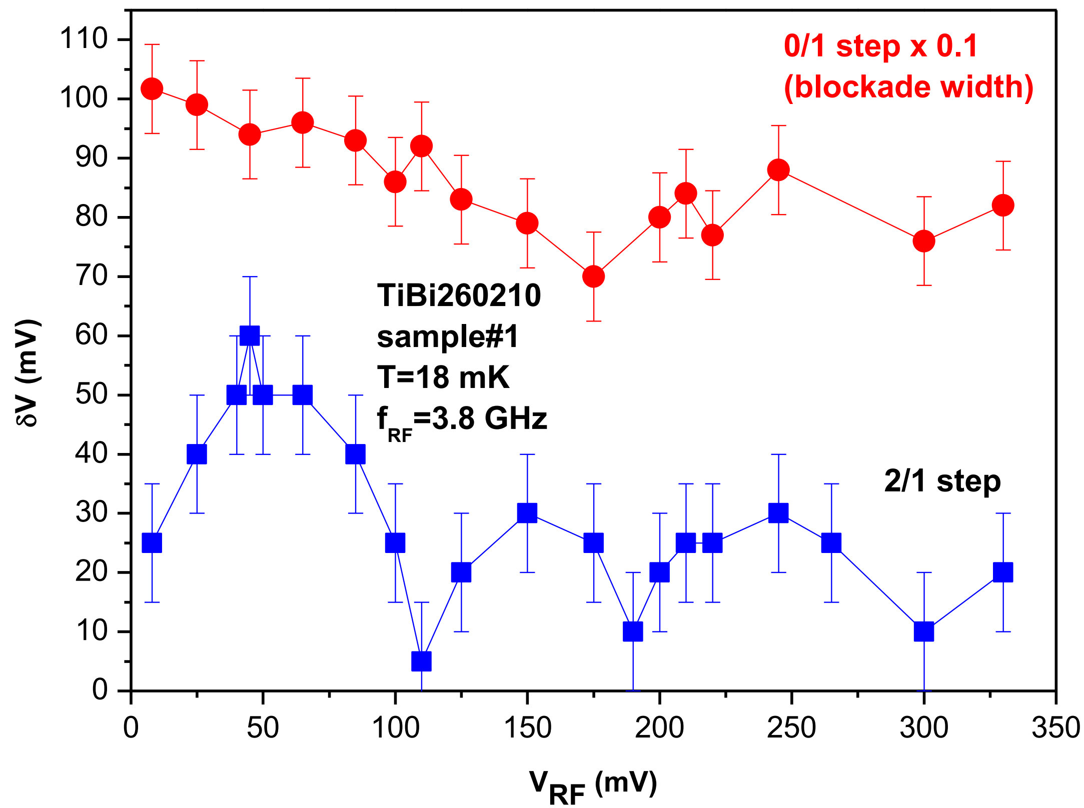
**With the increase of the RF frequency (= larger value of the current) the width of the steps increases, but the noise increases and (sub)harmonics appear.**




Red arrows correspond to the expectations for charge (2e) and the blue arrows – to single electron oscillations (e).

# Width of the current step vs. RF radiation amplitude



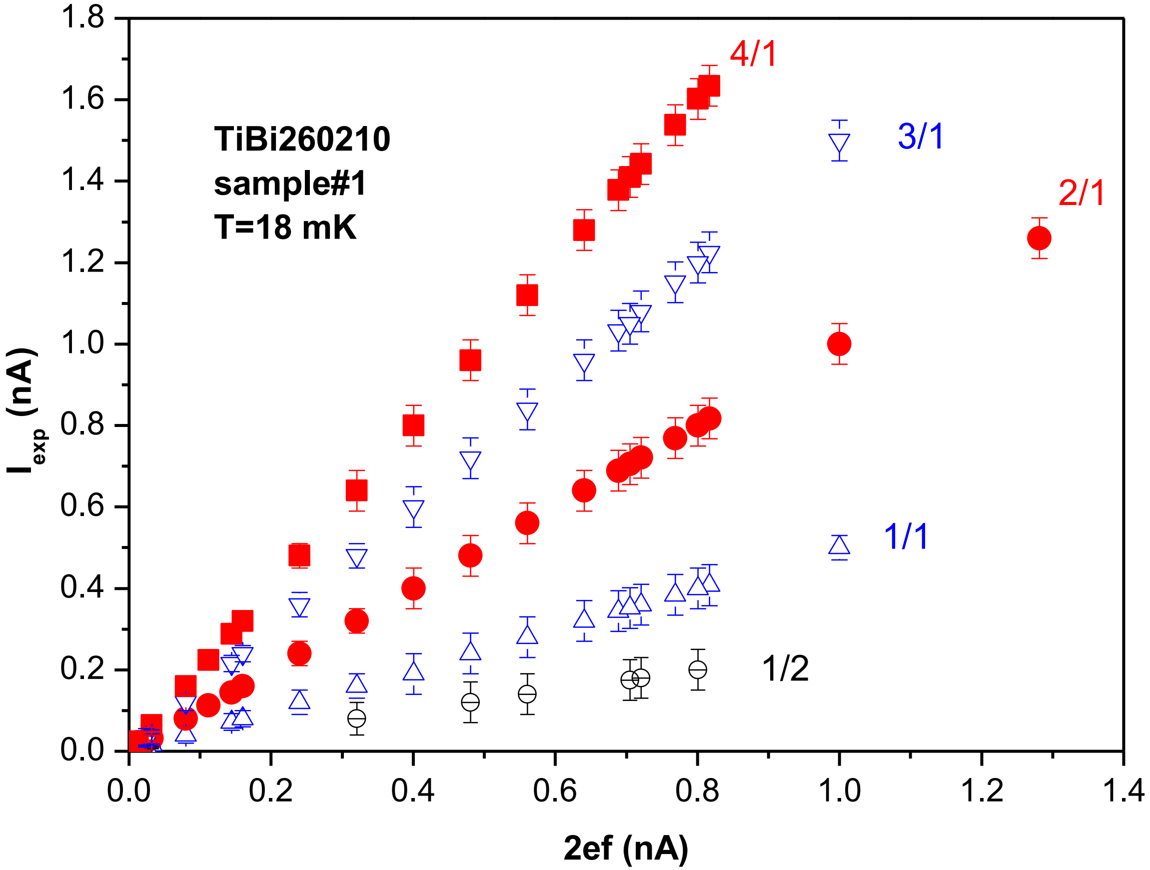

Width of the n-th step current step

$$\delta V_n \sim (-1)^n J_n(a)\sin(q),$$

where **q** – quasicharge, **a** – amplitude of the RF radiation, $\mathbf{J_n}$ – Bessel function

[*D. V. Averin and A. A. Odintsov, Fizika Nizkix Temperatur, 16, 16 (1990)*]

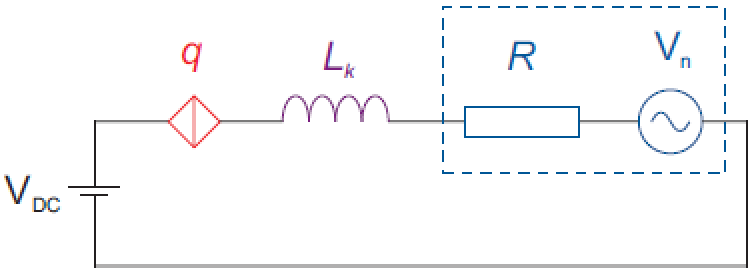
# Positions of the RF-induced steps




Red symbols correspond to Bloch steps (2e), blue – single electron (e), **black – 1st subharmonic** of single electron oscillations.

# Proof-of-principle demonstration of a quantum standard for electric current

# Simulated Bloch steps

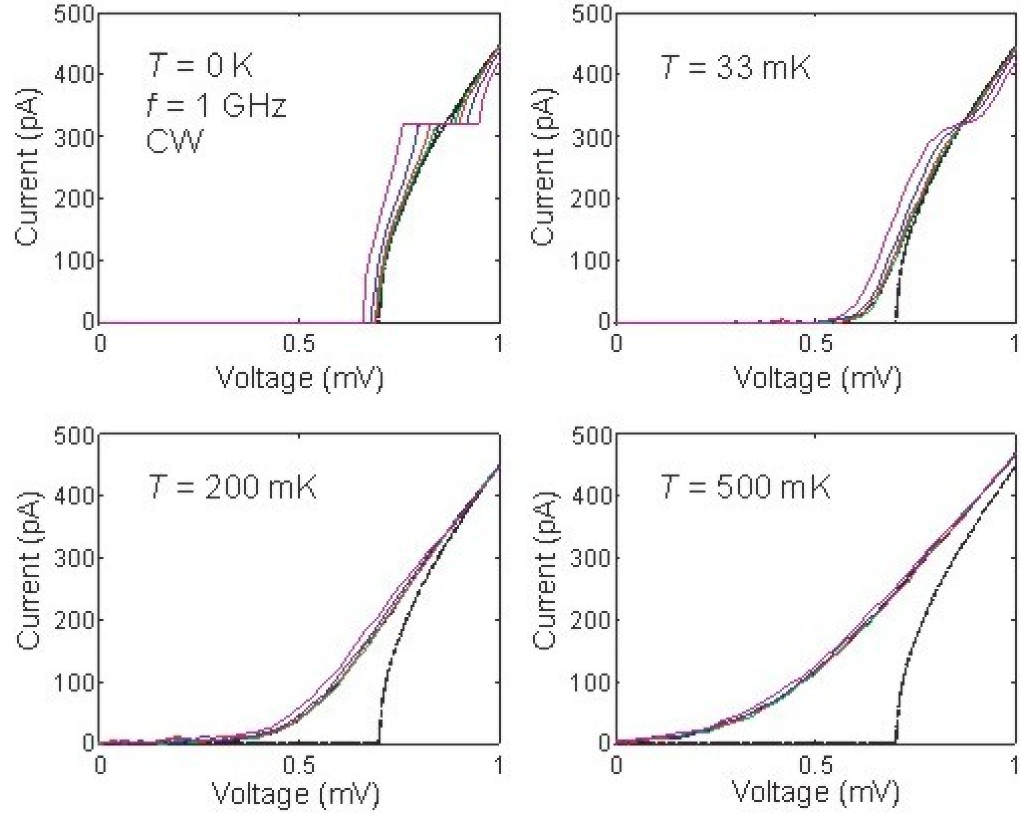



$$V(t) = V_c sin\left(\frac{2\pi q}{2e}\right) + L_k\frac{d^2q}{dt^2} + R\frac{dq}{dt} + V_n$$

where $V_n(t)$ is noise with rms amplitude

$$V_{n,rms} = \sqrt{4Rk_BT\delta_f}$$

at temperature T and bandwidth $\delta_f$, $I \equiv dq/dt$.




*Simulated I-V characteristics of a NbSi nanowire for a continuous-wave ac signal with various amplitudes and frequency 1 GHz at T= 0, 33, 200 and 500 mK.*

Sharpness of the Bloch steps depends on noise. At a finite current I the Joule heating of the resistor R increases the temperature T, and, hence, broadens the steps.

*C. H. Webster, J. C. Fenton, T. T. Hongisto, S.P. Giblin, A. B. Zorin, and P. A. Warburton, PRB 87 144510 (2013)*

# High impedance environment

*M. Watanabe, D. Haviland , PRL 86, 5120 (2001); PRB 67, 094505 (2003); M. Watanabe, PRB 69, 094509 (2004).*

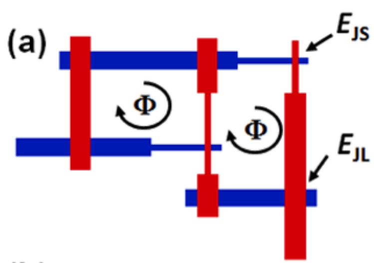
1D chain of SQUIDs

$$E_J = E_J(0)\cos\left(\frac{\phi}{\phi_0}\right)$$

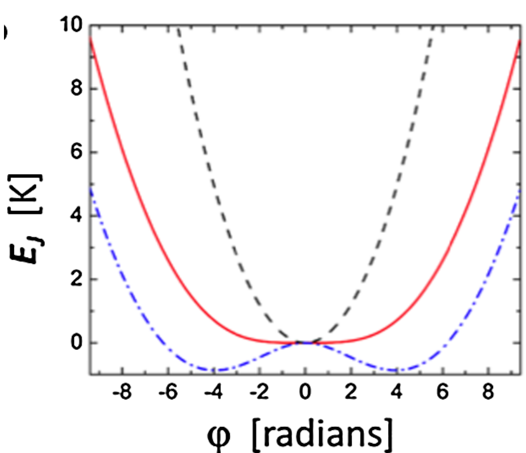
$$R_{dyn}(V \to 0) \equiv \frac{dV}{dI}(V \to 0) \sim 1/\cos\left(\frac{\phi}{\phi_0}\right)$$

$$\mathcal{L}_K = \left(\frac{d^2E_J}{d\phi^2}\right)^{-1} \sim 1/\cos\left(\frac{\phi}{\phi_0}\right)$$

At $\phi/\phi_0 \to \pi/2$ both $R_{dyn} \to \infty$ and $\mathcal{L}_K \to \infty$.



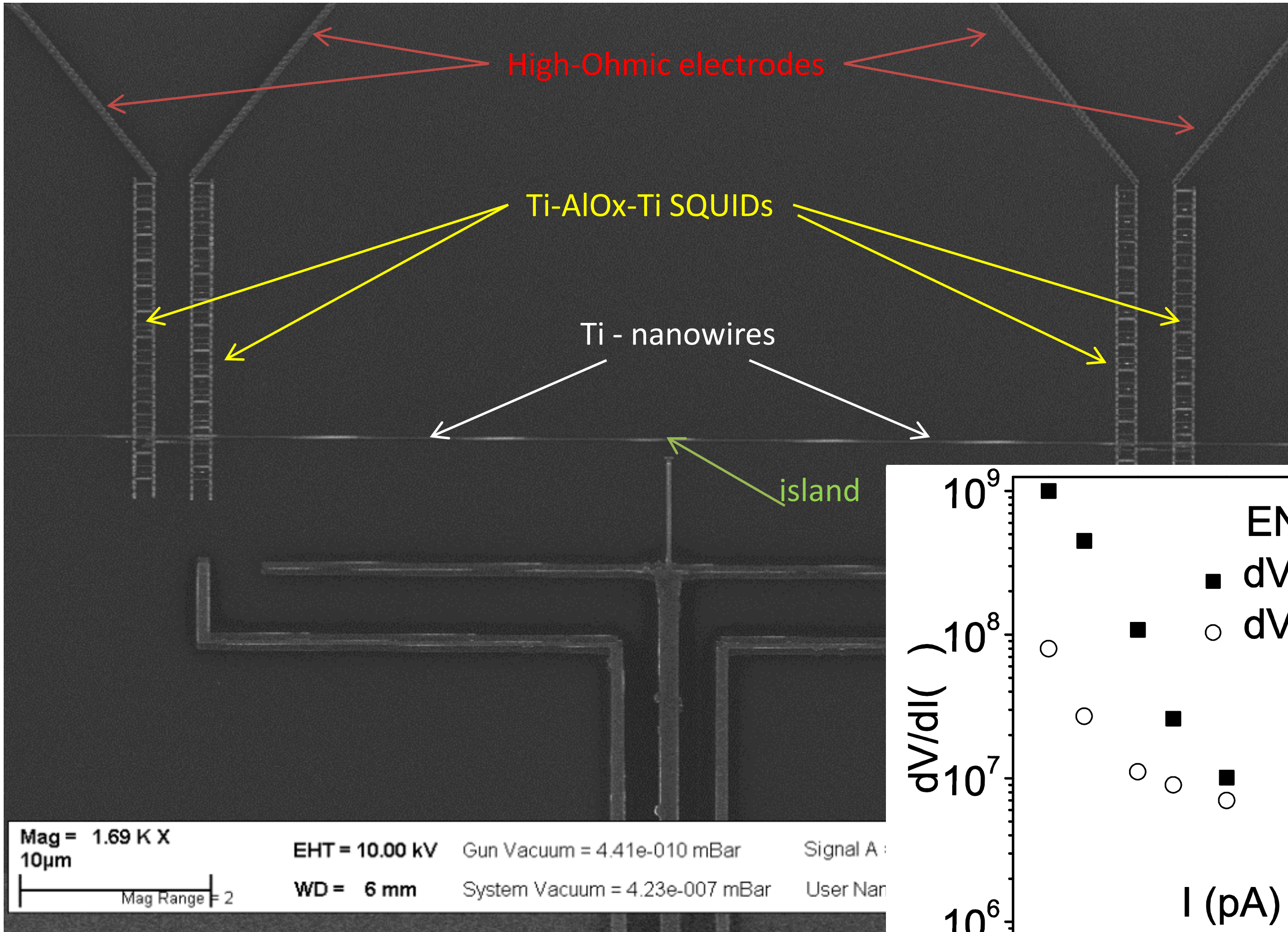

# Superinductance

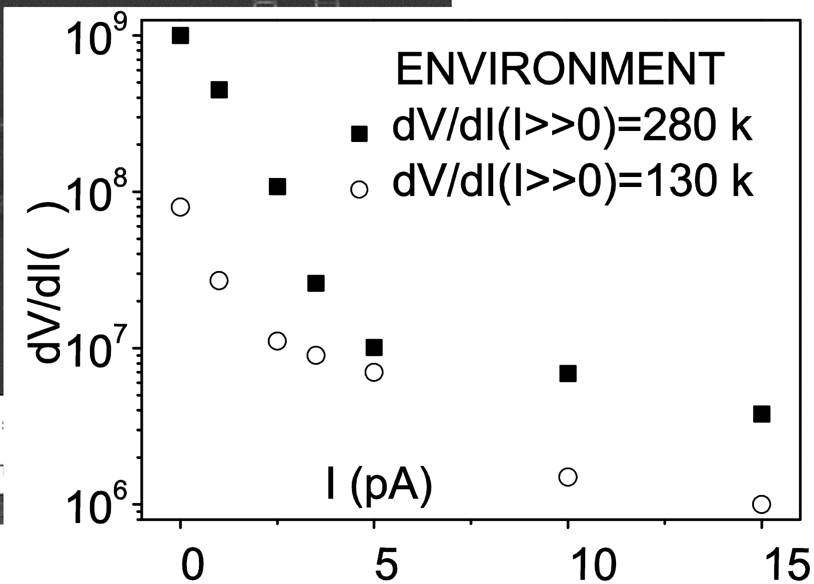



In more 'complicated' Josephson systems the $E_J(\phi)$ dependence can be strongy anharmonic. Within the locus of the anharmonicty the kinetic inductance can be made very large $\mathcal{L}_K \to \infty$.

*M. T. Bell, at. al. PRL 109, 137003 (2012) and N. A. Masluk, et.al. PRL 109, 137002 (2012)*

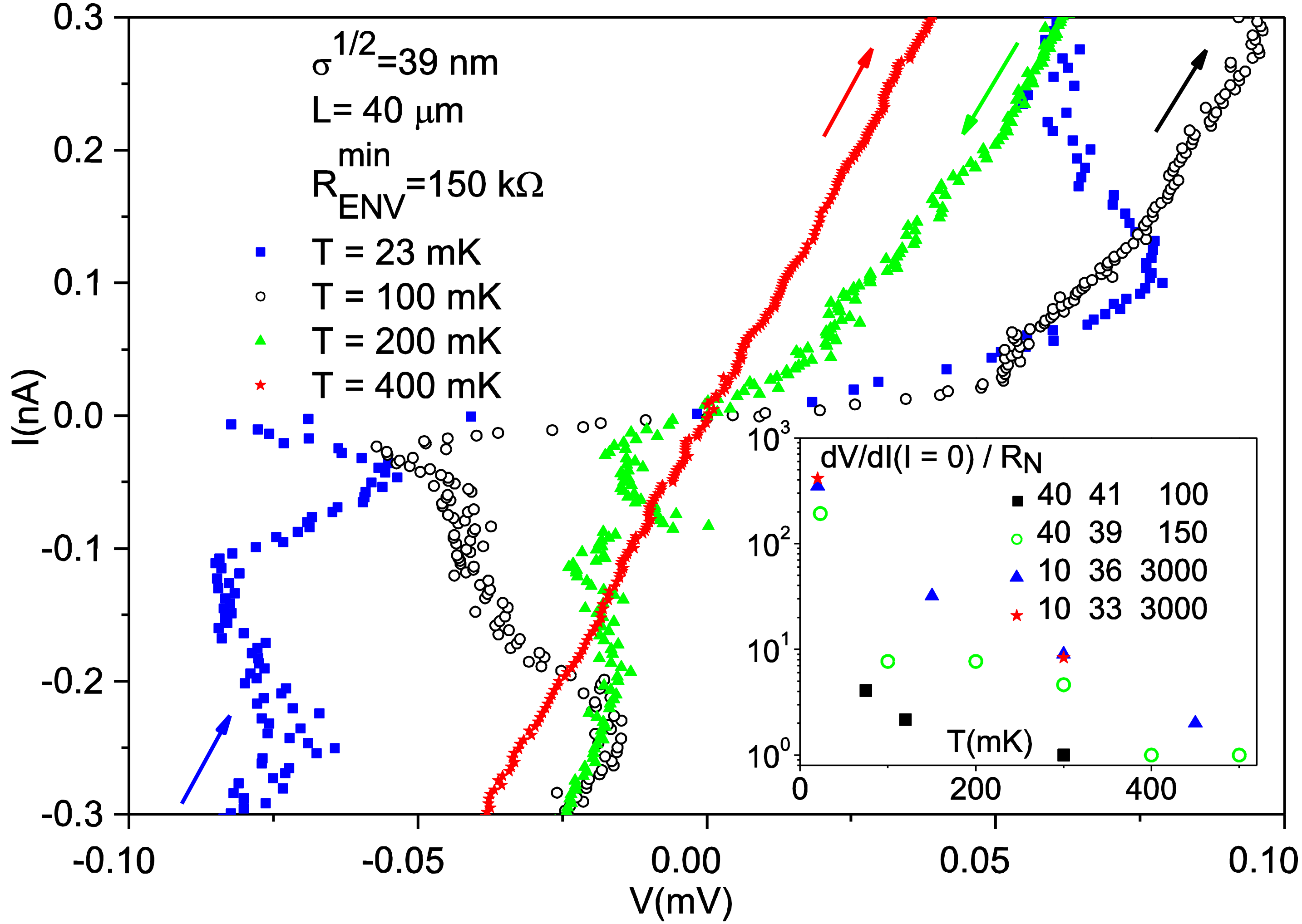
# Hybrid high impedance environment




High-Ohmic normal metal probes with $R_{probe}$ up to 500 KOhm and 1D array of SQUIDs.

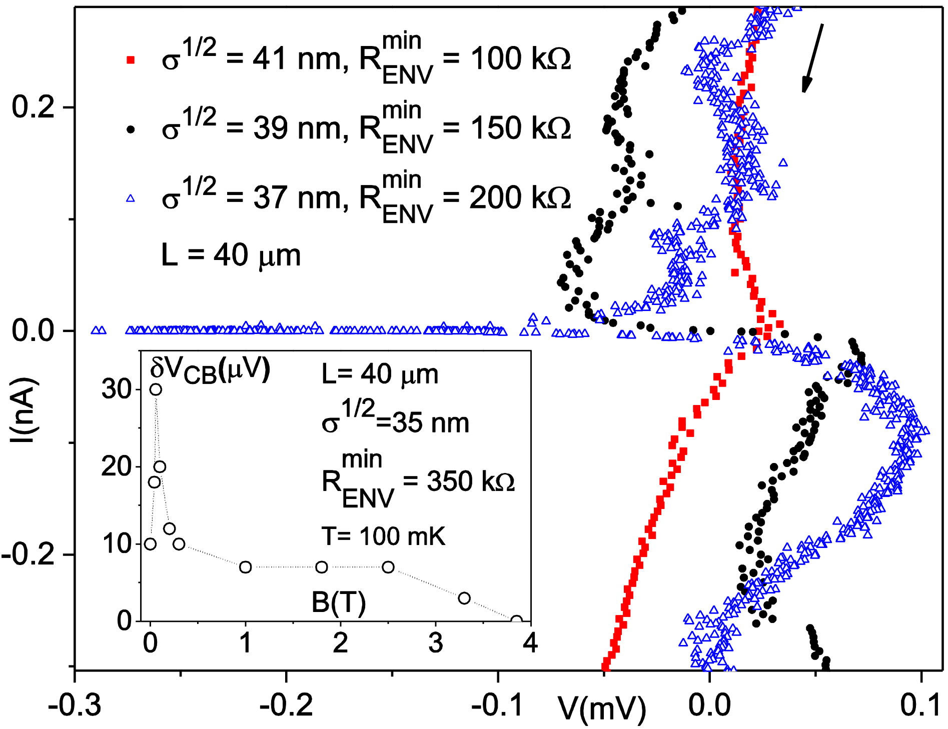
# Temperature dependence of the Coulomb blockade

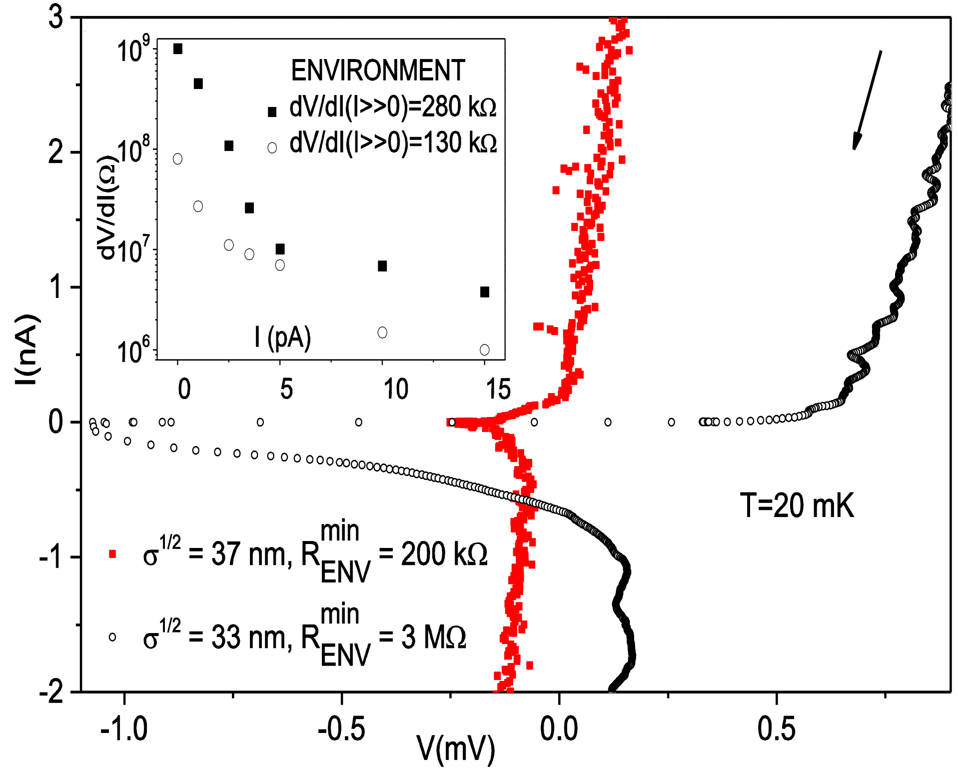



Inset: temperature dependencies of the zero-bias dynamic resistance of several nanowires, normalized by the normal state resistance $R_N$. Length $L$(μm), effective diameter $\sigma^{1/2}$(nm) and high-bias impedance (kΩ) of the environment $R_{ENV}$ are listed in the inset.

The insulating state (Coulomb blockade) disappears above $T_c$.

# Coulomb blockade in high impedance environment

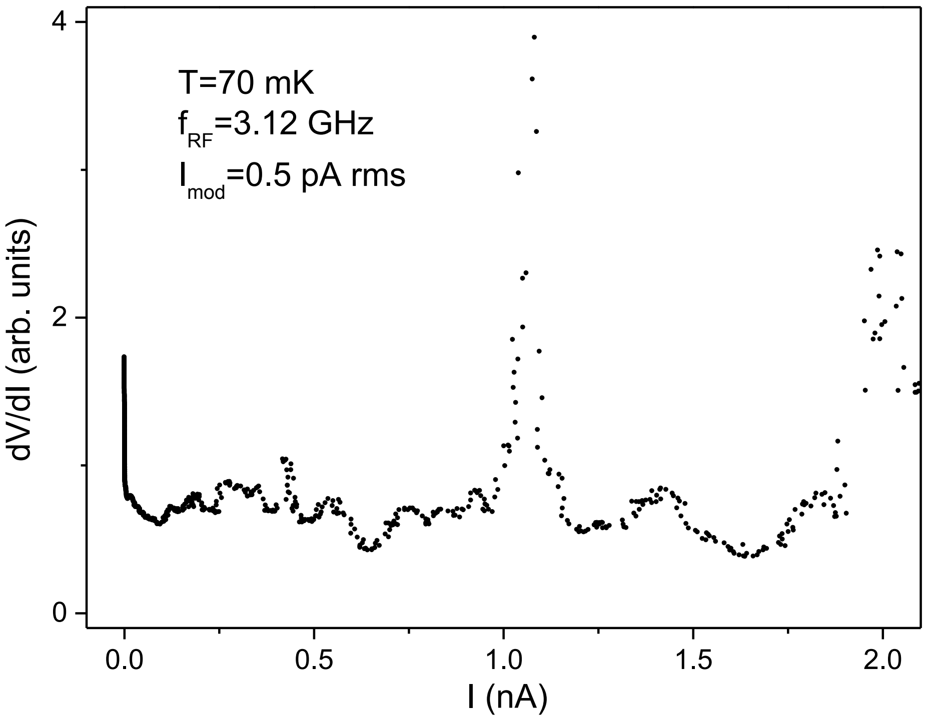


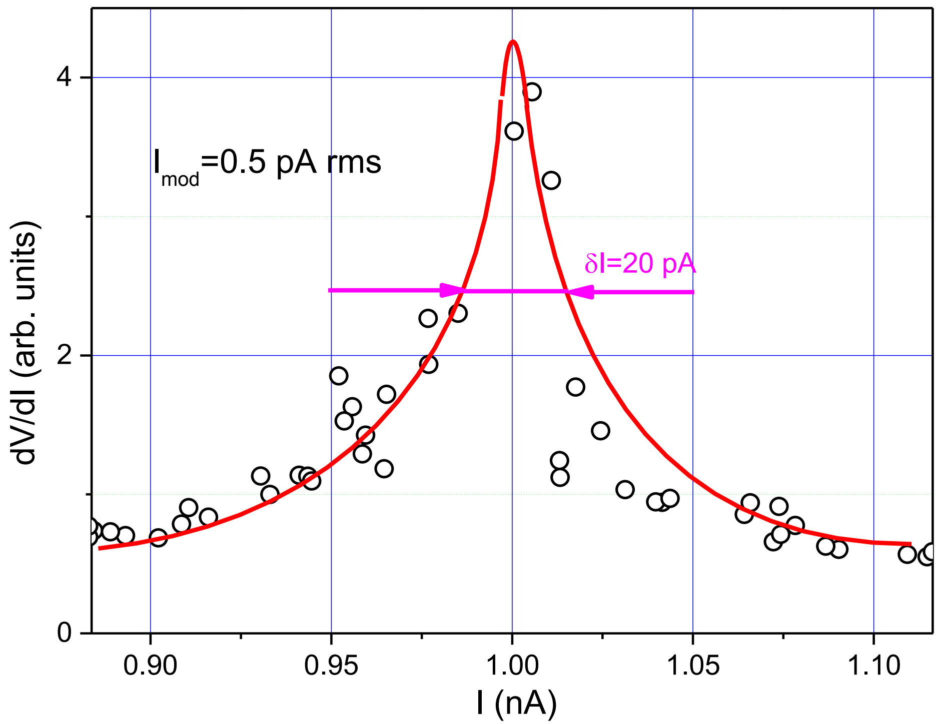




**Coulomb blockade vs. cross section σ. Inset: magnetic field dependence of the Coulomb gap.**

**Coulomb blockade vs. impedance of the environment $R^{min}_{ENV}$. Inset: dynamic resistance dV/dI at various biases for typical SQUID probes.**

# Width of the Bloch step




**Demonstrated accuracy is +/- 2%.**
**Further accuracy improovement is mandatory for practical metrology.**

# Conclusions on applications

Quantum phase slip phenomenon opens possibility for new applications dual to Josephson systems.

Thank you!