Multiplication of distributions and nonlinear problems in mathematical physics

J.F. Colombeau

I developed a nonlinear differential calculus on distributions after 1981 under support of Prof. L. Nachbin (Rochester and Rio de Janeiro) and L. Schwartz (Paris). About 1987 Y. Egorov informed me of similarities with Maslov theory. Now it is clear that this differential calculus and the developments of Maslov theory due to Danilov, Omel'yanov and Shelkovich complement each other. I will sketch some selected aspects of my work that could complement this last theory.

I will show that the proof given by L. Schwartz to support his claim on the impossibility to multiply distributions (1953) proves something else, namely: in any reasonable differential algebra containing the distributions the classical result

$$\int F(x)\phi(x)dx = 0 \forall \phi \in \mathcal{C}_c^\infty \Rightarrow F = 0$$

does not hold any longer. This does not cause any serious drawback and I could construct a differential algebra having optimal properties, see a survey in Bulletin of the AMS, 23, 1990, p. 251-268.

I will show how this differential algebra produces new formulas in physics that were verified qualitatively and quantitatively for strong collisions of elastic solids with definitive deformations and possible breakings, see Lecture Notes in Math 1532, Springer, 1992.

I will sketch unfinished applications to Quantum Field Theory and PDEs.